RSIM-BASED INTEGRITY MONITORING FOR DIFFERENTIAL GNSS

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Abstract

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The paper discusses the integrity monitoring for differential GNSS service provider stations that are built to the RSIM standard as recommended by the Radio Technical Commission For Maritime Services (RTCM). The main RSIM algorithm uses pseudorange residuals for failure detection. These residuals are not normalised to a common standard deviation, which makes the failure detection problem at hand differ from the standard algorithms in navigation literature. This paper introduces an analysis technique that can handle the RSIM algorithm by assessing its performance using conservative assumptions.

1. Introduction: the RSIM standard

The RSIM standard [RSIM] delineates the performance, functional, interface and environmental parameters for DGPS reference stations and integrity monitors, and has been written to encourage consistency among service providers of differential GPS (or GNSS) services. The RSIM standard complements [RTCM] that standardizes the communication between the differential service provider and the user and will be referred to as the RTCM standard, that defines RTCM messages. The RSIM standard defines a number of Reference Station/Integrity Monitoring (RSIM) messages (see Appendix B) that enable communications between the various equipment in the reference station (RS), integrity monitor (IM) and control station (CS) regardless of the manufacturer, see also Figure 1. The three main entities on the service-provider side are:

- Reference Station (RS): the reference station uses a reference receiver to generate differential GNSS corrections in accordance with [RTCM].
- Integrity Monitor (IM): the IM receives the RS broadcast and verifies the information content of the differential corrections. The IM provides positive feedback to the RS on a regular basis to indicate the system operates within specification. When an out-of-tolerance condition is detected, the IM generates alarms and notifies the RS that can undertake appropriate action.
- Control Station: the system is operated from the CS. It will receive status information on system performance and anomalies, and initiate corrective actions when required. It also sets parameter settings for both the RS and IM operation, including the threshold settings for alarm generation within the IM.

The RSIM standard is written for Marine Radiobeacons that broadcast their DGPS information using Minimum Shift Keying (MSK) modulation on a medium frequency, and is used by numerous manufacturers in the field. Except for the data link, the standard is not limited to these beacon systems, as is demonstrated by the successful implementation of the RSIM standard within the Eurofix system [Willigen98], an integrated radionavigation and communication system, which has been proposed and developed by Delft University of Technology. Loran-C or Chayka stations are upgraded to broadcast low-speed differential DGPS over ranges up to 1,000 km.

1.1 Scope and organization of this paper

The paper is organised as follows. First of all, the integrity monitoring algorithms within the IM are briefly discussed. At the core is the algorithm that detects errors in the pseudorange domain, and the paper is limited to a detailed analysis of this algorithm. A method to analyse its performance and to set its tuning is introduced. The paper then ends with some concluding remarks.

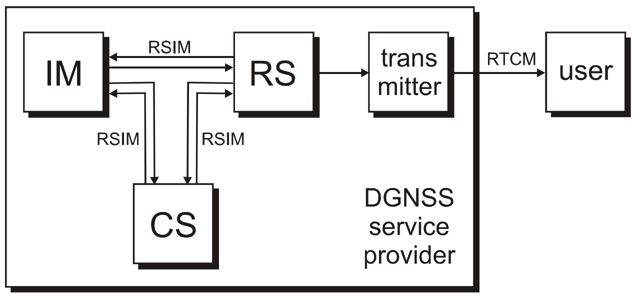

Figure 1. The building blocks of a differential GNSS service.

2. IM failure detection algorithms

The RSIM standard describes some error detection algorithms that are part of the IM. Two algorithms will be discussed briefly here: pseudorange error detection and position domain error detection. The former detection is at the core of the integrity providence for the whole system. When this check works well, it is highly unlikely that a position domain error will ever be detected; the position domain check should therefore be seen as a backup only and is not discussed in detail here. Its performance analysis would be comparable to that of standard RAIM algorithms that has been discussed amply in existing literature [Brown92, Kelly96, Leva96].

2.1 Pseudorange Error Detection

The main task of the IM is to check the integrity of the pseudorange corrections (PRC's) that the RS broadcasts. The IM measures its own GPS pseudoranges and corrects these with the received corrections. The corrected ranges are compared to the ranges that are computed using the IM's known position. The IM has to account both for its own clock bias and the bias which resides in the PR corrections, due to the RS clock bias, in determining the difference between the true range to the satellite and the corrected-measured range. After correcting for these biases, an error is obtained for each correction-measurement combination, which is called the PR residual.

An alarm condition occurs if the absolute value of the PR Residual exceeds the threshold as set by RSIM message 16. If this alarm condition persists for a period longer than the PR Residual Observation Interval (also in RSIM message 16), an alarm is raised and sent to the RS using RSIM message 20. When an insufficient number of satellites are in view, the check cannot be performed reliably and an alarm is provided that the reference station has become unmonitored.

2.2 Position Domain Error Detection

As a final overall check, the accuracy of the DGPS positioning is evaluated by the IM. The IM calculates its position using the GPS observables and DGPS corrections as if it were a user, that is, without using knowledge on its own position. The calculated position is compared to the known IM position. Horizontal position error alarms are evaluated by computing the horizontal radial error. An alarm is raised when the horizontal position error exceeds the threshold for a period exceeding the Horizontal Position Error Observation Interval. Both the threshold and the observation interval are set using RSIM message 16. When the geometry is insufficient to calculate a reliable horizontal position, because the HDOP is too large, the check cannot be performed reliably and an alarm is provided that the reference station has become unmonitored.

3. Pseudorange error sources and model

3.1 Error sources

The pseudoranges that are measured contain errors from a variety of sources. Some of these errors will be the same for the RS and the IM while others will differ for each individual receiver-antenna pair, and will therefore differ for the RS and the IM. Assuming that the RS and the IM are situated closely to each other, the most important common error sources are atmospheric delays, satellite clock errors and ephemeris prediction errors. Because these errors are common to the RS and the IM, they will only play a minor role in the integrity monitoring process. It will be assumed that these errors are slowly varying and therefore have a bias-like (rather than a random noise) character. In that case, their influence is essentially zero. This is not as obvious as it might seem at first sight. Because of their unknown clock-biases, both the RS and the IM receivers will have to estimate their clocks from the measurement data. The common errors listed above will influence these estimates, but possibly to a different extend, as the RS and the IM might have different satellites in view, or might use different weighting factors for satellites based on different receiver noise levels. However, because the bias in the clock estimate for the RS will show up in all corrections that it broadcasts (as the next section will show) the IM cannot distinguish between its own clock bias and the RS clock-bias. Because of this, the common errors will only influence the IM clock bias estimate, which is removed before error detection is performed. Therefore, the error detection process is not influenced, even if different satellites or different weightings are used.

The non-common error sources that *will* be important are mainly receiver noise and multipath. The sum of receiver noise and multipath on the pseudorange measurement to satellite i will be denoted by $e_{RS,i}$ and $e_{IM,i}$ for the RS and the IM respectively. For more notations and conventions, see Appendix A.

The error model that is used in this paper is the so-called slippage of the mean model. It is assumed that the non-common error sources are normally distributed. In the case of normal operation, the mean of this distribution should be (close to) zero. In case of failure, the mean becomes non-zero (biased) and the probability of large errors increases. In mathematical terms, this can be written as:

$$\begin{array}{ll} normal\ operation: & e_{RS,i} \sim N(0,\sigma_{RS,i}^2) \\ failure: & e_{RS,i} \sim N(\mu_{RS,i},\sigma_{RS,i}^2) \end{array} \quad (1)$$

As in general little knowledge on the behaviour of the system in the case of failures is available, the magnitude of the failure-induced bias $\mu_{RS,i}$ is not specified.

4. The pseudorange residual

In this section, the pseudorange residual (which will often be called just 'the residual' for brevity) that forms the basis of the IM based error detection algorithms is discussed in more (mathematical) detail, before the actual performance analysis of the RSIM algorithm can be described in the next chapter.

The residuals are the difference between the differentially corrected measured ranges and the known actual ranges, taking clock biases of the IM and the RS receivers into account. The first part of the residual generation process is the computations of the corrections at the RS. These are therefore addressed first, before discussing the processing at the IM itself.

4.1 Reference station correction generation

The raw (that is, uncorrected for clock-biases) pseudorange correction for satellite *i* is the difference between the actual distance to the satellite $R_{RS,i}$ and the pseudorange measured by the RS. It contains three sources of error: all common and non-common error sources mentioned above, and the clock bias of the receiver. As the common errors are neglected, the error in the raw corrections equals:

$$\Delta\vec{\rho}_{RS} = \begin{bmatrix} R_{RS,1} - \rho_{RS,1} \\ \vdots \\ R_{RS,N_{RS}} - \rho_{RS,N_{RS}} \end{bmatrix} = \begin{bmatrix} e_{RS,1} \\ \vdots \\ e_{RS,N_{RS}} \end{bmatrix} + \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} \cdot b_{RS} = \vec{e}_{RS} + \vec{h} \cdot b_{RS} \quad (2)$$

with the error vector zero mean normally distributed with a (diagonal) covariance matrix Σ_{RS}:

$$\vec{e}_{RS} \sim N(\vec{0},\Sigma_{RS}) \quad (3)$$

It is assumed that the RS removes the clock bias on an epoch-by-epoch basis and estimates it using a weighted least squares algorithm. The clock bias estimate can thus be written as:

$$\hat{b}_{RS} = \vec{w}_{RS}^T \Delta\vec{\rho}_{RS}\ \text{, with}\ \vec{w}_{RS}^T = (\vec{h}^T \Sigma_{RS}^{-1} \vec{h})^{-1} \vec{h}^T \Sigma_{RS}^{-1} = [w_{RS,1} \quad \cdots \quad w_{RS,N_{RS}}] \quad (4)$$

The clock estimation error thus equals:

$$\hat{b}_{RS} - b_{RS} = \vec{w}_{RS}^T \vec{e}_{RS} \quad (5)$$

After correcting for the clock by subtracting the estimated clock bias, the error in the pseudorange correction that is broadcast by the RS becomes:

$$\vec{e}_{prc} = \Delta\vec{\rho}_{RS} - \vec{h}\hat{b}_{RS} = \vec{e}_{RS} + \vec{h}(\vec{w}_{RS}^T \vec{e}_{RS}) \quad (6)$$

The elements of this vector can be written as:

$$e_{prc,i} = \vec{w}_{RS,i}'^T \vec{e}_{RS}\ \text{, with}\ \vec{w}_{RS,i}'^T = [-w_{RS,1} \cdots (1 - w_{RS,i}) \cdots - w_{RS,N_{RS}}] \quad (7)$$

The covariance matrix of the resulting pseudorange correction errors now reads:

$$\Sigma_{prc} = \left[\vec{w}'_{RS,1} \quad \cdots \quad \vec{w}'_{RS,N_{RS}} \right]^T \Sigma_{RS} \left[\vec{w}'_{RS,1} \quad \cdots \quad \vec{w}'_{RS,N_{RS}} \right] \quad (8)$$

Due to the estimation of the clock, the corrections have become correlated while their standard deviations have decreased as can be seen from the element-by-element representation of (8):

$$\Sigma_{prc}[i][j] = \begin{cases} (1-w_{RS,i})^2\sigma_{RS,i}^2 + \sum_{k=1,k\neq i}^{N_{RS}} w_{RS,k}^2\sigma_{RS,k}^2 & (i=j) \\ -w_{RS,i}(1-w_{RS,i})\sigma_{RS,i}^2 - w_{RS,j}(1-w_{RS,j})\sigma_{RS,i}^2 + \sum_{k=1,k\neq i,k\neq j}^{N_{RS}} w_{RS,k}^2\sigma_{RS,k}^2 & (i\neq j) \end{cases} \quad (9)$$

An estimate of the standard deviation of the corrections is send along with the corrections and is called the User Differential Range Error (UDRE).

4.2 Integrity monitor residual generation

At the integrity monitor, raw pseudorange corrections can be computed in a similar fashion as in the RS. The difference between these corrections and the RS computed corrections are called the raw (that is, not corrected for IM clock biases) pseudorange correction residuals:

$$\overrightarrow{rr} = \Delta\vec{\rho}_{IM} - \overrightarrow{prc} = \vec{h}\cdot b_{IM} + \vec{e}_{rr} \quad (10)$$

with the noise component equal to:

$$\vec{e}_{rr} = \vec{e}_{IM} - \vec{e}_{prc} \quad (11)$$

The covariance matrix of these raw residuals easily follows as:

$$\Sigma_{rr} = \Sigma_{IM} + \Sigma_{prc} \quad (12)$$

The clock bias at the IM will be removed in a way similar to that at the RS using these raw residuals. The IM bias is thus estimated as:

$$\hat{b}_{IM} = \vec{w}_{IM}^T\,\overrightarrow{rr} = b_{IM} + \vec{w}_{IM}^T\vec{e}_{rr} \text{ with } \vec{w}_{IM}^T = (\vec{h}^T\Sigma_{rr}^{-1}\vec{h})^{-1}\vec{h}^T\Sigma_{rr}^{-1} = [w_{IM,1} \quad \cdots \quad w_{IM,N_{IM}}] \quad (13)$$

Note that the optimal weights are determined from the standard deviations of all individual raw residuals, which include local noise sources as well as the noise in the corrections. These standard deviations can be estimated at the IM from the receiver noise level and from the UDRE that is sent along with the corrections, although this might lead to sub-optimal values, as both might contain (estimates) of some of the common error sources.

After applying the estimated IM clock bias to correct the raw pseudorange residuals, the (clock corrected) pseudorange residual for the (correction of) the i^{th} pseudorange correction, that is to be used for error detection purposes, is obtained at the IM as:

$$r_i = \vec{w}_{IM,i}'^T\vec{e}_{rr} \text{ with } \vec{w}_{IM,i}'^T = [-w_{IM,1}\cdots(1-w_{IM,i})\cdots - w_{IM,N}] \quad (14)$$

The variance of the (clock corrected pseudorange) residual equals:

$$\Sigma_r = \left[\vec{w}_{IM,1}' \quad \cdots \quad \vec{w}_{IM,N_{RS}}'\right]^T \Sigma_{rr}\left[\vec{w}_{IM,1}' \quad \cdots \quad \vec{w}_{IM,N_{RS}}'\right] \quad (15)$$

An important observation from (15) is that this covariance matrix in general will neither be diagonal, nor have all equal diagonal entries. One can thus conclude that the residuals are correlated and possibly have different standard deviations.

4.3 Residual based error detection

When there is a bias in the difference of the measurements to satellite *i* at the RS and the IM caused by a failure condition at the RS the mean of $e_{RS,i}$ becomes non-zero, say μ_i. Due to the correlation effect that the estimation of the clock bias introduces, all residuals are influenced. Using the expressions (7), (11) and (14) and substituting the mean error of the corrections:

$$\vec{\mu}_{RS,i} = [0 \cdots 0 \quad \underset{i}{\mu_i} \quad 0 \cdots 0]^T \tag{16}$$

one finds the following mean for the residual vector:

$$\vec{\mu}_{r,i} = \mu_i [-w_i \cdots - w_i \quad \underset{i}{(1-w_i)} \quad -w_i \cdots - w_i]^T \text{ with } w_i = -(w_{RS,i} + w_{IM,i}) + \sum_{k=1}^{N_{IM}} w_{RS,k} w_{IM,k} \tag{17}$$

Error detection can therefore be based on a test between the following hypotheses:

$$\begin{aligned} &H_0: \textit{ normal operation, } \vec{r} \sim N(\vec{0}, \Sigma_{rr}) \\ &H_i: \textit{ failure in } i^{th} \textit{ correction, } \vec{r} \sim N(\vec{\mu}_{r,i}, \Sigma_{rr}) \end{aligned} \tag{18}$$

Sum of squared errors

In navigation literature, there are two standard failure detection algorithms known. The first is based on the normalised norm of the vector of the residuals errors that has a non-central chi-squared distribution with N_{IM}-1 degrees of freedom, see for example [Brown92]:

$$SSE = \vec{r}^T \Sigma_r^{-1} \vec{r} \sim \chi^2 (N_{IM} - 1, \lambda) \text{ with non-centrality } \lambda = \vec{\mu}_{r,i}^T \Sigma_r^{-1} \vec{\mu}_{r,i} \tag{19}$$

The test between the 'normal operation' and 'failure' hypotheses is equivalent to a test on the noncentrality parameter of the distribution of the *SSE*:

$$\begin{aligned} &H_0\,(no\ error): \ \lambda = 0 \\ &H_1\,(error): \qquad \lambda > 0 \end{aligned} \tag{20}$$

As the test statistic has an expected value that grows linearly with the noncentrality parameter

$$E\{SSE\} = \lambda + N_{IM} - 1 \tag{21}$$

it is indeed suitable to perform such a test. The final decision on the detection of a failure will be taken by comparing *SSE* to a threshold. If the threshold is exceeded, the 'no error' hypothesis is supposed to be sufficiently unlikely, and a failure is assumed:

$$\begin{aligned} &SSE \le SSE_{threshold} \Rightarrow conclude\ H_0\ is\ correct \\ &SSE > SSE_{threshold} \Rightarrow conclude\ H_i\ is\ correct \end{aligned} \tag{22}$$

Maximum absolute normalised residual

An alternative to the *SSE* based test has been described in [Kelly96], and is called the maximum residual technique. In this paper, the method will be referred to as the *maximum absolute normalised residual* or *MNAR* algorithm as it uses the following test statistic:

$$MNAR = \max_i |r_i| / \sigma_{r,i} \tag{23}$$

Because of the maximisation, the distribution of the *MNAR* is mathematically intractable. Yet, it is clear that the *MNAR* will grow with the value of μ_i and can be used to distinguish between the hypotheses just like the *SSE* in (22). Unlike the *SSE* based technique discussed above, the element for which the absolute residual is largest indicates immediately which of the corrections is most likely to have been in failure.

RSIM: Maximum absolute residual

In the RSIM tests, the residual has to exceed a threshold over an observation period rather than just at one particular sample to actually cause an alarm. Due to the usage of carrier-phase smoothing, which is a standard technique for differential reference stations, the errors in the residuals will be highly correlated over a relatively long period [Shively00]. It will therefore be assumed that the RSIM algorithms can only take one independent sample over the observation period and the algorithm can be analysed as if it were instantaneous. The IM will thus raise an alarm for correction *i* as soon as the corresponding pseudorange residual exceeds a threshold.

The RSIM standard assumes a separate hypothesis test for each individual correction/residual. However, as can be seen from the specification of RSIM message 20, only a single correction can be flagged to be failing simultaneously. Although this is not affirmed by the standard, it has been assumed here that in the case that multiple residuals simultaneously grow beyond the threshold, the largest of those residuals is the most likely to correspond to an erroneously large correction, and the PRN of this particular correction is the one that is sent to the RS for further action. It is thus assumed that the test of interest uses the following test-statistic:

$$MAR = \max_i |r_i| \tag{24}$$

A failure is detected as soon as $MAR > MAR_{threshold}$ and is contributed to the correction that corresponds to the largest absolute residual. The algorithm that has been described in the RSIM standard is therefore strikingly similar to the *MNAR* algorithm. There is, however, an important difference: the failure detection test in the RSIM standard does not normalise the residual to unit standard deviation, as both the *SSE* and *MNAR* algorithms do (by dividing the residual by its standard deviation). This means there is only a single threshold, set by RSIM message 16, that is used for detection of biases in residuals with different noise levels, which complicates analysis.

5. Performance analysis of the RSIM algorithm

As has been described in section 4.3, in case of a detected failure, the satellite that is excluded from the solution by means of an RSIM message 20 is the one of which the residual exceeds a certain threshold $MAR_{threshold}$ and the residuals from other satellites. This is illustrated graphically in Figure 2 where for simplicity only two residuals are shown. It is assumed that the first residual is biased, making the joint distribution of the two residuals shift from zero along the line $((1-w_1)\mu_1, w_1\mu_1)$. There are three cases of interest to characterise performance: missed detections, wrong identification and false alarms. In the white area all residuals are small and no error is detected despite the bias. This is therefore the missed detection area. In the dark grey area, satellite 2 is incorrectly identified as the erroneous satellite (wrong identification). Correct identification takes place when the residuals are in the light grey area.

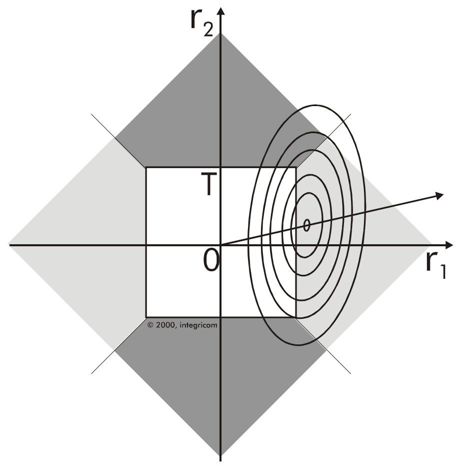

Figure 2. The different areas that indicate the probabilities of missed detection (white), correct (light grey) and erroneous (dark grey) failure identification.

In the presence of an error, tuning should be such that the probability of missed detection is small, which requires a small threshold *T*. Simultaneously, the probability of wrong identification should be small, which is promoted by a large threshold. As a side effect of failure-detection, in case there is no error, satellites can be identified as erroneous due to noise, which is called ‘false detection’. This degrades system continuity and its occurrence should be kept to a minimum, which requires a large threshold as well.

The setting of the threshold thus balances the amount of false detections and wrong identifications with the number of missed detections. As can be seen from Figure 2, calculation of the probabilities on these events in the most general case require integration of the mutual distribution of the N_{IM} residuals over the three differently coloured regions. This mutual distribution depends on a number of parameters that are time varying and not available to the system operator, such as the standard deviations of the residual, the weights used in the clock bias estimations, and the magnitude of the failure induced bias. To set a threshold, it will therefore be unavoidable to make assumptions on these parameters. In the following, it is assumed that the threshold has been set to a value *T*, which could for example based on the assumptions on all equal weights and set such, that the probability of false alarms obeys the system requirements:

$$MAR_{threshold} = T \tag{25}$$

The main goal of the remaining section is to show how the probabilities of false alarm, missed detection and wrong identification can be determined as a function from *T* and the failure induced bias, and reducing the integration of the N_{IM} dimensional distribution of the residual vector to a one or bidimensional distribution only. In all cases, every step should guarantee conservative performance results. For simplicity of notation, it will be assumed without loss of generality that the failure occurs in the correction indexed by *i*=1 and that the induced bias is positive, that is, $\mu_1 > 0$.

5.1 False alarms

The probability of false alarms is relatively easy to assess. It is the probability that at least one of the residuals exceeds the threshold *in the absence of RS failure induced biases*:

$$P_{FA} = P(\max_i |r_i| > T \mid no\ bias) \tag{26}$$

Using the (first) Bonferroni inequality [Kelly96], one can find the following conservative upperbound on this false alarm probability:

$$P_{FA} \le \sum_{i=1}^{N} P(|r_i| > T \mid no\ bias) \tag{27}$$

Another upperbound that could be used and that is somewhat tighter is the Joshi upperbound [Joshi72] that is in fact the probability that would be obtained in the absence of correlation between the residuals. It is given by

$$P_{FA} \le 1 - \prod_{i=1}^{N} P(|r_i| < T \mid no\ bias) \tag{28}$$

In both cases, the bound can be found from the distributions of the separate scalar residuals rather than a multidimensional mutual distribution.

5.2 Missed detections

The probability of missed detection is the probability that, while there is a bias in satellite 1, none of the residuals exceeds the threshold:

$$P_{MD}(\mu_1) = P(\max_i |r_i| < T \mid \mu_1) \tag{29}$$

An conservative upperbound to (29) can be obtained by replacing the maximum residual by any other residual, of which the first is the most suited as it will be influenced most by the bias and thus gives the tightest bound:

$$P_{MD}(\mu_1) \le P(|r_1| < T \mid \mu_1) \tag{30}$$

Again, the bound reduces the need for use of a multidimensional distribution to a scalar one.

5.3 Misidentification and wrong exclusion

The misidentification case is by far the most complicated one. The probability that a wrong residual is the largest equals can be used as a conservative upperbound, as in reality misidentification only occurs when the largest residual also exceeds the threshold *T*:

$$P_{MI}(\mu_1) \le P(|r_1'| < \max_i |r_i| \mid given\ \mu_1) \tag{31}$$

A further upperbound to (31) is readily found by summing the probabilities that the first residual is smaller than each of the others (applying what is called the 'union bound'):

$$P_{MI}(\mu_1) \le \sum_{i=2}^{N} P(|r_1| < |r_i| \mid given\ \mu_1) = \sum_{i=2}^{N} [1 - P(|r_1| > |r_i| \mid given\ \mu_1)] \tag{32}$$

The computation is now brought down from N to 2 dimensions, and the computation of N-1 probabilities of the form:

$$P_{1i} = P(|r_1| > |r_i| \; given \; \mu_1) \tag{33}$$

Defining the vector of difference and sum of the two residuals involved in (33) as:

$$\vec{q}_{1i} = \begin{bmatrix} d_{1i} \\ s_{1i} \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} r_1 \\ r_i \end{bmatrix} \tag{34}$$

one can write:

$$|r_1| > |r_i| \Leftrightarrow [(r_1 > 0) \wedge \vec{q}_{1i} \in \; first \;\; quadrant] \vee [(r_1 < 0) \wedge \vec{q}_{1i} \in third \;\; quadrant] \tag{35}$$

An underbound to probability P_{1i} from (33) (leading to an upperbound of P_{MI}) can be found readily by neglecting the (unlikely) that correct identification occurs while the first residual is negative (remember that $\mu_1 > 0$), and only consider $r_1 > 0$, and thus:

$$P_{1i} \geq P(\vec{q}_{1i} \in \; first \;\; quadrant) \tag{36}$$

The probability of misidentification (31) can thus be expressed conservatively in terms of the distribution of (34), that obeys, writing σ_i for the standard deviation of r_i:

$$\vec{q}_{1i} \sim N\left(\begin{bmatrix} \mu_1 \\ (1-2w_1)\mu_1 \end{bmatrix}, \Sigma_{q,1i} \right), \Sigma_{q,1i} = \begin{bmatrix} \sigma_1^2 + \sigma_i^2 & \sigma_1^2(1-2w_1) - \sigma_i^2(1-2w_i) \\ \sigma_1^2(1-2w_1) - \sigma_i^2(1-2w_i) & (1-2w_1)^2\sigma_1^2 + (1-2w_i)^2\sigma_i^2 + 4\sum_{k=2,k\neq i}^{N} w_k^2\sigma_k^2 \end{bmatrix} \tag{37}$$

A bound to P_{MI} can thus be found from the binormal distribution of (34). Note that when the standard deviations σ_i of all residuals would be equal, and as a results all weights w_i are equal, the diagonal elements of (37) vanish. This implies that the distributions of the two elements of (34) become statistically independent, which would further reduce the problem to a scalar distribution case.

6. User performance

With the expressions of the previous section, it is possible to assess the performance obtained by the RSIM integrity monitoring algorithms in a conservative manner. This could for example be done using the idea of minimal detectable biases (MDB), which is a standard technique in navigation literature [Leva96]. One could tune the algorithms by setting the threshold T such that the maximum allowed false alarm probability obeys the requirements using (28) or (27). Given T it can be assessed for which (minimum) value of the failure-induced bias (MDB) the requirements for the probability of missed detection and misidentification are obeyed, using the upperbounds to (29) and (31). The frequency with which biases occur determines the frequency the user performance is affected by biases with the size of at most the MDB. This is the bias a user will have to be able to live with at the given frequency.

7. Concluding remarks

The paper has discussed the integrity monitoring algorithms from the RSIM standard and shown how the most important performance parameters can be computed. The main RSIM algorithm uses pseudorange residuals with different noise levels for failure detection, but has to compare them all to the same threshold. This not only prevents the use of standard algorithm performance assessment techniques from navigation literature, but necessarily leads to suboptimal performance as well. The author expresses his hope that future version of the standard will take this into account.

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Appendix A: Notations and conventions

A hat (^) denotes the estimated value of a parameter. Many parameters listed below are also used in vector form. The symbol used for the covariance-matrices of such vectors is Σ. Many of the RS parameters have similar counterparts for the IM that are not listed separately.

PR: pseudorange
PRC: pseudorange correction

$R_{RS,i}$	range of satellite i	P_{MI}	probability of misidentification
$\rho_{RS,i}$	PR of satellite i	P_{FA}	probability of false alarm
$\Delta\rho_{RS,i}$	raw PRC of satellite i	P_{MD}	probability of missed detection
$e_{RS,i}$	RS specific errors in PR of satellite i	μ_i	magnitude of failure-induced bias in PR correction i
$\sigma_{RS,i}$	standard deviation of $e_{RS,i}$	$\vec{\mu}_{RS,i}$	vector of biases in all corrections
b_{RS}	RS clock bias	$\vec{\mu}_{r,i}$	vector of biases in all residuals
$\sigma_{\Delta\rho_{RS,i}}$	standard deviation of $\Delta\rho_{RS,i}$	$\vec{q}_{1i}$	vector with sum and difference of residual 1 and i
$e_{prc,i}$	error in broadcasted PRC for satellite i	P_{1i}	probability residual 1 exceeds residual i in absolute value
prc_i	broadcasted PRC for satellite i	T	threshold
N_{RS}	number of satellites the RS has in view	$\vec{h}$	vector of ones
N_{IM}	number of satellites the IM has in view and has received corrections for	$w_{RS,i}$	weights to derive RS clock estimate from the raw corrections
r_i	PR residual to satellite i	$\vec{w}'^T_{RS}$	weights to derive error in broadcast corrections from the measurement
rr_i	raw (not clock-bias corrected) PR residual to satellite i	$w_{IM,i}$	weights to derive IM clock estimate from the raw residuals
$e_{rr,i}$	error in raw PR residual to satellite i	$\vec{w}'^T_{IM}$	weights to derive PR residuals from error in raw residuals
$\sigma_{r,i}$	standard deviation of $e_{r,i}$	w_i	weight in the relation between bias in residual and PR correction

Appendix B: RSIM Messages

Version 1 of the RSIM standard defines twenty-five different RSIM messages. Only a few of these messages are relevant within the scope of this paper. All messages contain a standard header for identification, a varying number of fields that contain all information, and a checksum field to detect errors in the message communication. This appendix lists all relevant fields from the RSIM messages 16, 17 and 20 and the parameters they contain.

A.1. RSIM message 17 – IM Alarm Thresholds and Intervals

RSIM message 16 is sent from the CS to the IM and sets all parameters that are relevant for the error detection algorithms. It contains thresholds and intervals for each of the alarms that the IM can trigger. In general, a fault condition must exist for the associated interval before an alarm is generated. If any interval is zero, an instantaneous violation of the threshold will generate an alarm. The list of parameters that are relevant for this paper is shown in the table and correspond to the 'low HDOP', the 'high horizontal position error' and 'high pseudorange residual' alarms.

Field	Parameter	Value
8	low number of satellites tracked threshold	integer
9	low number of satellites tracked interval	in seconds
a	HDOP threshold	in meters
b	HDOP observation interval	in seconds
c	horizontal position error threshold	in meters
d	horizontal position error observation interval	in seconds
e	pseudorange residual threshold in meters	in meters
f	pseudorange residual observation interval	in seconds

A.2. RSIM message17 – Integrity Monitor Alarms

RSIM message 17 is sent from the IM to the control station to provide detailed information alarms when they occur or cease. The parameters for these alarms are set by RSIM message 16.

Field	Parameter	Value
1	UTC Time	UTC time
7	Number Of Satellites Tracked Alarm	Z (zero) / L (low) / A (acceptable)
8	High HDOP Alarm	H (high) / A (acceptable)
9	Horizontal Position Alarm	H (high) / A (acceptable)
a	High Pseudorange Residual Alarm	H (high) / A (acceptable)

A.3. RSIM message 20 – Integrity Monitor System Feedback

RSIM message 20 is sent on a routine basis to the RS to provide positive feedback. In case of the detection of a position failure by the position domain error detection, the position flag is set to 'error', while under high HDOP conditions, it is set to "unmonitored". In case of a large pseudorange residual, message 20 gives individual satellite PRN trouble information, which may enable the RS to take corrective action. Only one PRN can be flagged bad at a time. Corrective action could include discontinuing use of that PRN, or resetting tracking loops. Users can be informed to discontinue use of this particular PRN by a special RTCM message.

Field	Parameter	Value
2	Position Flag	0 (OK) / 1 (Error) / 2 (Unmonitored)
3	Pseudo Range Residual Flag	0-32 (PRN of detected satellite)