RSIM-Based Integrity Monitoring for DGNSS

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BIOGRAPHY

Bastiaan Ober's areas of experience include the influence of multipath on GPS positioning, carrier phase differential GPS, ambiguity resolution and integrity monitoring, especially for aviation applications. He is currently working as a Ph.D. student doing research on integrity design and analysis of integrated navigation systems for safety critical applications.

ABSTRACT

The paper discusses the integrity monitoring for differential GNSS service provider stations that are built to the RSIM standard as recommended by the Radio Technical Commission For Maritime Services (RTCM). The main RSIM failure detection algorithm is based on the use of pseudorange residuals, which are not normalized to a common standard deviation and therefore differs from standard failure detection algorithms in navigation literature. This paper discusses a conservative analysis technique that computes the probabilities that the RSIM algorithm fails to assess the absence or presence of failure-induced biases correctly.

1. INTRODUCING THE RSIM STANDARD

The RSIM standard [RSIM] delineates the performance, functional, interface and environmental parameters for DGPS reference stations and integrity monitors, and has been written to encourage consistency among service providers of differential GPS (or GNSS) services. The RSIM standard defines a number of Reference Station/Integrity Monitoring (RSIM) messages (see Appendix B) that enable communications between the various equipment in the reference station (RS), integrity monitor (IM) and control station (CS) regardless of the manufacturer, see also Figure 1. The three main entities on the service-provider side are:

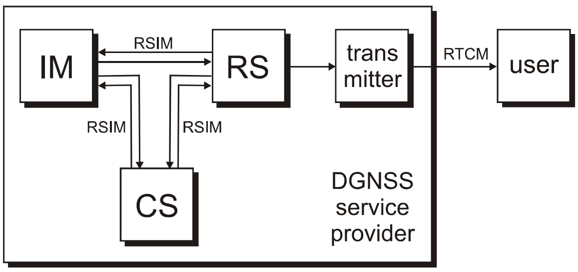

Figure 1. The building blocks of a differential GNSS service, and the corresponding communication protocols.

- Reference Station (RS): the reference station uses a reference receiver to generate differential GNSS corrections that are broadcast to the user (and the IM) in accordance with the protocol from [RTCM]

- Integrity Monitor (IM): the IM receives the RS broadcast and verifies the information content of the differential corrections. The IM provides positive feedback to the RS on a regular basis to indicate the system operates within specification. When an out-of-tolerance condition is detected, the IM generates alarms and notifies the RS that can undertake appropriate action

- Control Station: the system is operated from the CS. It will receive status information on system performance and anomalies and initiate corrective actions when required. The CS also sets parameter settings for both the RS and IM operation, including the threshold settings for alarm generation within the IM.

The RSIM standard complements the RTCM standard [RTCM], which standardizes the communication between the differential service provider and the user by defining a number of RTCM messages. It was originally written for Marine Radiobeacons that broadcast their DGPS information using Minimum Shift Keying (MSK) modulation on a medium frequency, and is used for that purpose by numerous manufacturers in the field. Except for the data link, application of the standard is not limited to these beacon systems, as is demonstrated by the successful implementation of the RSIM standard within the Eurofix system [Willigen98], an integrated radionavigation and communication system, which has been proposed and developed by Delft University of Technology. Within Eurofix, Loran-C or Chayka stations are upgraded to broadcast low-speed differential DGPS over ranges up to 1,000 km over a low-frequency data link.

The paper briefly discusses the main integrity monitoring mechanism within the IM. A method to analyze and tune its performance is then introduced, based on a conservative assessment of the probabilities of false alarm, missed detection and wrong identification. In this area, the paper extends the results of [Ober01], on which it is largely based, with a new and simpler upper bound. Also, the translation to the user's performance is sketched. The paper then ends with some concluding remarks.

2. IM FAILURE DETECTION ALGORITHMS

The RSIM standard describes some error detection algorithms that are part of the IM. It does so in a partly implicit manner by defining a number of messages and explaining the meaning and use of the content of these messages.

Only one failure detection algorithm will be discussed in this paper: the pseudorange failure detection, as this algorithm forms the core of the integrity providence. When this detection mechanism works well, it is highly unlikely that a position domain failure, that is checked as well, is ever going to be detected; the position domain check that is also part of the standard should therefore be seen as a backup test. This view is also supported by the message structure: information on a position failure is only sent to the CS (using message 17) but not to the RS. This means that no automatic and immediate action is foreseen for the position domain failure, which implies that the contribution to system integrity is minimal. It is therefore not discussed further.

2.1 Pseudorange Failure Detection

The main task of the IM is to check the integrity of the pseudorange corrections (PRC's) that the RS broadcasts. When an insufficient number of satellites are in view, the check cannot be performed reliably and an alarm is to be provided that the reference station has become unmonitored.

The IM measures its own GPS pseudoranges and corrects these with the received corrections. The corrected ranges are compared to the ranges that are computed using the IM's known position. The IM has to account both for its own clock bias and the bias which resides in the PR corrections, due to the RS clock bias, in determining the difference between the true range to the satellite and the corrected-measured range. After correcting for these biases, an error is obtained for each correction-measurement combination, which is called the PR residual or just residual.

An alarm condition occurs if the absolute value of the residual exceeds the threshold as set by RSIM message 16. If this alarm condition persists for a period longer than the residual Observation Interval (also in RSIM message 16), an alarm is raised and sent to the RS using RSIM message 20. Due to the usage of carrier-phase smoothing, which is a standard technique for differential reference stations, the errors in the residuals will be highly correlated over a relatively long period [Shively00]. It will therefore be assumed in this paper that the RSIM algorithms can only take one independent sample over the observation period and the algorithm can be analyzed as if it were instantaneous. The IM will thus raise an alarm for correction i as soon as the corresponding pseudorange residual exceeds a threshold.

2.2 RSIM Messages

Version 1 of the RSIM standard defines twenty-five different RSIM messages. Future standards will contain more messages, but the draft version 1.1 (May 2000) currently contains no changes to the messages that are relevant within the scope of this paper: the messages 16 and 20. All messages contain a standard header for identification, a varying number of fields that contain all information, and a checksum field to detect errors in the message communication.

2.2.1 Message 16 – Failure Detection Settings

RSIM message 16 is sent from the control station to the IM and contains values that the IM should use for all parameters that are relevant for the failure detection algorithms. It contains thresholds and intervals for each of the alarms that the IM can trigger. In general, a fault condition must exist for the associated interval before an alarm is generated. If any interval is zero, an instantaneous violation of the threshold will generate an alarm. The list of parameters that are relevant for this paper is shown in Table 2. The parameters correspond to the 'low HDOP', the 'high horizontal position error' and 'high pseudorange residual' alarms.

Field	Parameter	Value
8	low number of satellites tracked threshold	integer
9	low number of satellites tracked interval	in seconds
a	HDOP threshold	in meters
b	HDOP observation interval	in seconds
c	horizontal position error threshold	in meters
d	horizontal position error observation interval	in seconds
e	pseudorange residual threshold in meters	in meters
f	pseudorange residual observation interval	in seconds

Table 1. The contents of RSIM Message 16

2.2.2 Message 20 – IM System Feedback

RSIM message 20 is sent on a routine basis to the RS to provide feedback on the status of the failure detection at the IM. In case of the detection of a position failure by the position domain error detection, the position flag is set to 'error', while under high HDOP conditions, it is set to "unmonitored". Most importantly, message 20 gives individual satellite PRN trouble information in case of a large pseudorange residual, which enables the RS to take corrective. Corrective action could include discontinuing use of that PRN, or resetting tracking loops. Users can be informed to discontinue use of this particular PRN by a special RTCM message. Only one bad PRN can be flagged at a time. *It will be assumed in this paper that the PRN that is flagged is the one with the largest residual, although this is not explicitly stated in the standards.*

Field	Parameter	Value
2	Position Flag	0 (OK)/1 (failure)/2 (unmonitored)
3	PR Residual Flag	0-32 (PRN of detected satellite)

Table 2. The contents of RSIM Message 20

3. PSEUDORANGE NOISE MODEL

3.1 Error sources

The pseudoranges that are measured contain errors from a variety of sources. Some of these errors will be the same for the RS and the IM while others will differ for each individual receiver-antenna pair, and will therefore differ for the RS and the IM. Assuming that the RS and the IM are situated closely to each other, the most important common error sources are atmospheric delays, satellite clock errors and ephemeris prediction errors. Because these errors are common to the RS and the IM, they will only play a minor role in the integrity monitoring process. It will be assumed that these errors are slowly varying and therefore have a bias-like (rather than a random noise) character. In that case, their influence is essentially zero. This is not as obvious as it might seem at first sight. Because of their unknown clock-biases, both the RS and the IM receivers will have to estimate their clocks from the measurement data. The common errors listed above will influence these estimates, but possibly to a different extend, as the RS and the IM might have different satellites in view, or might use different weighting factors for satellites based on different receiver noise levels or other measures of the measurement variance. However, because the bias in the clock estimate for the RS will show up in all corrections that it broadcasts (as the next section will show) the IM cannot distinguish between its own clock bias and the RS clock-bias. Because of this, the common errors will only influence the IM clock bias estimate, which is removed before error detection is performed. Therefore, the error detection process is not influenced, even if different satellites or different weightings are used.

The most important non-common error sources are receiver noise and multipath. The sum of receiver noise and multipath on the pseudorange measurement to satellite i will be denoted by $e_{RS,i}$ and $e_{IM,i}$ for the RS and the IM respectively. The error model that is used in this paper is the so-called slippage of the mean model. It is assumed that the non-common error sources are normally distributed. In the case of normal operation, the mean of this distribution should be (close to) zero. In case of failure, the mean becomes non-zero (biased) and the probability of large errors increases. In mathematical terms, this can be written as:

$$\begin{aligned} &\textit{normal operation}: e_{RS,i} \sim N(0, \sigma^2_{RS,i}) \\ &\textit{failure}: \qquad e_{RS,i} \sim N(\mu_{RS,i}, \sigma^2_{RS,i}) \end{aligned} \tag{1}$$

As in general little knowledge on the behavior of the system in the case of failures is available, the magnitude of the failure-induced bias $\mu_{RS,i}$ is not specified in the model.

4. THE PSEUDORANGE RESIDUAL

Before the actual performance analysis of the RSIM algorithm can be described, the pseudorange residual (which will often be called just ‘the residual’ for brevity) that forms the basis of the IM based error detection algorithms needs to be discussed in more (mathematical) detail.

The residuals are the difference between the differentially corrected measured ranges and the known actual ranges, taking clock biases of the IM and the RS receivers into account. The first part of the residual generation process is the computations of the corrections at the RS. These are therefore addressed first, before discussing the processing at the IM itself.

4.1 Reference station correction generation

The raw (that is, uncorrected for clock-biases) pseudorange correction for satellite i is the difference between the actual distance to the satellite $R_{RS,i}$ and the pseudorange measured by the RS. It contains three sources of error: all common and non-common error sources mentioned above, and the clock bias of the receiver, written as b_{RS}. As the common errors are neglected, the error in the N_{RS} raw corrections equals:

$$\Delta\vec{\rho}_{RS} = \begin{bmatrix} e_{RS,1} \\ \vdots \\ e_{RS,N_{RS}} \end{bmatrix} + \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} \cdot b_{RS} = \vec{e}_{RS} + \vec{h} \cdot b_{RS} \tag{2}$$

with the error vector zero mean normally distributed with a (diagonal) covariance matrix Σ_{RS}:

$$\vec{e}_{RS} \sim N(\vec{0}, \Sigma_{RS}) \tag{3}$$

It is assumed that the RS removes the clock bias on an epoch-by-epoch basis and estimates it using a weighted least squares algorithm. The estimated clock bias estimate can thus be written as:

$$\hat{b}_{RS} = \vec{w}^T_{RS} \Delta\vec{\rho}_{RS} \tag{4}$$

with the vector of weights defined as:

$$\vec{w}^T_{RS} = (\vec{h}^T \Sigma^{-1}_{RS} \vec{h})^{-1} \vec{h}^T \Sigma^{-1}_{RS} = [w_{RS,1} \quad \cdots \quad w_{RS,N_{RS}}] \tag{5}$$

The clock estimation error thus equals:

$$\Delta b_{RS} = \hat{b}_{RS} - b_{RS} = \vec{w}^T_{RS} \vec{e}_{RS} \tag{6}$$

After correcting for the clock by subtracting the estimated clock bias, the error in the pseudorange correction that is broadcast by the RS becomes:

$$\vec{e}_{prc} = \Delta\vec{\rho}_{RS} - \vec{h}\hat{b}_{RS} = \vec{e}_{RS} + \vec{h}\cdot\Delta b_{RS} \qquad (7)$$

The elements of this vector can be written as the following function of the error vector elements:

$$e_{prc,i} = \vec{w}'^{T}_{RS,i}\vec{e}_{RS} \qquad (8)$$

with the 'adapted weights' defined by:

$$\vec{w}'^{T}_{RS,i} = [-w_{RS,1}\cdots(1-w_{RS,i})\cdots - w_{RS,N_{RS}}] \qquad (9)$$

Using the error propagation law, the covariance matrix of the resulting pseudorange correction errors becomes:

$$\Sigma_{prc} = \left[\vec{w}'_{RS,1} \quad \cdots \quad \vec{w}'_{RS,N_{RS}}\right]^T \Sigma_{RS}\left[\vec{w}'_{RS,1} \quad \cdots \quad \vec{w}'_{RS,N_{RS}}\right] (10)$$

Due to the estimation of the clock, the corrections have become correlated, while their standard deviations have generally decreased as can be seen from the element-by-element representation of (10) [Ober01]. An estimate of the standard deviation of the corrections is send along with the corrections and is called the User Differential Range Error (UDRE).

4.2 Integrity monitor residual generation

At the integrity monitor, raw pseudorange corrections can be computed in a similar fashion as in the RS. The difference between these corrections and the RS computed corrections are called the raw (not corrected for IM clock biases) pseudorange correction residuals:

$$\vec{r}_{raw} = \vec{h}\cdot b_{IM} + \vec{e}_r \qquad (11)$$

with the noise component equal to the difference of the errors on the corrections from the RS and the internal noise of the IM:

$$\vec{e}_r = \vec{e}_{IM} - \vec{e}_{prc} \qquad (12)$$

Because the noise sources are assumed to be independent, the covariance matrix of these raw residuals easily follows as:

$$\Sigma_{raw} = \Sigma_{IM} + \Sigma_{prc} \qquad (13)$$

in which the covariance matrix of the IM noise contributions contains the variances on the N_{IM} different IM measurements on its diagonal:

$$\Sigma_{IM} = diag(\sigma^2_{IM,1},\ldots,\sigma^2_{IM,N_{IM}}) \qquad (14)$$

The clock bias at the IM will be removed in a way similar to that at the RS using these raw residuals. The IM bias is thus estimated as:

$$\hat{b}_{IM} = \vec{w}^T_{IM}\vec{r}_{raw} = b_{IM} + \vec{w}^T_{IM}\vec{e}_r \qquad (15)$$

with

$$\vec{w}^T_{IM} = (\vec{h}^T\Sigma^{-1}_{raw}\vec{h})^{-1}\vec{h}^T\Sigma^{-1}_{raw} = [w_{IM,1} \quad \cdots \quad w_{IM,N_{IM}}] \quad (16)$$

Note that the optimal weights are determined from the standard deviations of all individual raw residuals, which include local noise sources as well as the noise in the corrections. These standard deviations can be estimated at the IM from the receiver noise variances and from the UDRE that is sent along with the corrections. Note that this might lead to sub-optimal values, as both might contain traces of the noise on common error sources.

After applying the estimated IM clock bias to correct the raw pseudorange residuals, the (clock corrected) pseudorange residual for the (correction of) the i^{th} pseudorange correction, that is to be used for error detection purposes, is obtained at the IM as:

$$r_i = \vec{w}'^{T}_{IM,i}\vec{e}_r \qquad (17)$$

with

$$\vec{w}'^{T}_{IM,i} = [-w_{IM,1}\cdots(1-w_{IM,i})\cdots - w_{IM,N}] \qquad (18)$$

The variance of the (clock corrected) residual equals:

$$\Sigma_r = \left[\vec{w}'_{IM,1} \quad \cdots \quad \vec{w}'_{IM,N_{RS}}\right]^T \Sigma_{raw}\left[\vec{w}'_{IM,1} \quad \cdots \quad \vec{w}'_{IM,N_{RS}}\right] (19)$$

An important observation from (19) is that this covariance matrix in general will neither be diagonal, nor have all equal diagonal entries. One can thus conclude that:

- the residuals are correlated
- the residuals have different variances

4.3 Residual based error detection

When there is a bias in the difference of the measurements to satellite i at the RS and the IM caused by a failure condition at the RS the mean of $e_{RS,i}$ becomes non-zero, say μ_i. When the mean error of the corrections contains a bias μ_i in the i^{th} measurement it has the following form:

$$\vec{\mu}_{RS,i} = [0\cdots0 \quad \underset{i}{\mu_i} \quad 0\cdots0]^T \qquad (20)$$

due to the correlation effect introduced by estimation of the clock bias, all residuals are influenced. Using the expressions (9), (12) and (18) one finds the following mean for the residual vector:

$$\vec{\mu}_{r,i} = \mu_i\,[-w_i\cdots - w_i \quad \underset{i}{(1-w_i)} \quad -w_i\cdots - w_i]^T \qquad (21)$$

in which

$$w_i = -(w_{RS,i} + w_{IM,i}) + \sum_{k=1}^{N_{IM}} w_{RS,k}w_{IM,k} \qquad (22)$$

Error detection can therefore be based on a test between the following hypotheses:

$$\begin{aligned} &H_0 : \textit{normal operation}, \ \vec{r} \sim N(\vec{0}, \Sigma_r) \\ &H_i : \textit{failure in } i^{th} \textit{ correction}, \ \vec{r} \sim N(\vec{\mu}_{r,i}, \Sigma_r) \end{aligned} \qquad (23)$$

In navigation literature, there are two standard failure detection algorithms known. The first and best known of these algorithms is based on the normalized norm of the vector of the residuals errors that has a non-central chi-squared distribution with N_{IM}-1 degrees of freedom and non-centrality λ, see for example [Brown92]:

$$SSE = \vec{r}^T \Sigma_r^{-1} \vec{r} \ \sim \chi^2 (N_{IM} - 1, \lambda), \ \lambda = \vec{\mu}_{r,i}^T \Sigma_r^{-1} \vec{\mu}_{r,i} \qquad (24)$$

The test between the ‘normal operation’ and ‘failure’ hypotheses is equivalent to a test on the non-centrality parameter of the distribution of the *SSE*:

$$\begin{aligned} &H_0 \, (\textit{no error}): \ \lambda = 0 \\ &H_1 \, (\textit{error}): \qquad \lambda > 0 \end{aligned} \qquad (25)$$

As the test statistic has an expected value that grows linearly with the noncentrality parameter

$$E\{SSE\} = \lambda + N_{IM} - 1 \qquad (26)$$

it is indeed suitable to perform such a test. The final decision on the detection of a failure will be taken by comparing *SSE* to a threshold. If the threshold is exceeded, the ‘no error’ hypothesis is supposed to be sufficiently unlikely, and a failure is assumed:

$$\begin{aligned} &SSE \le SSE_{threshold} \Rightarrow \textit{conclude } H_0 \textit{ is correct} \\ &SSE > SSE_{threshold} \Rightarrow \textit{conclude } H_i \textit{ is correct} \end{aligned} \qquad (27)$$

An alternative to the *SSE* based test has been described in [Kelly96], and is called the maximum residual technique. In this paper, the method will be referred to as the *maximum absolute normalized residual* or *MNAR* algorithm as it uses the following test statistic:

$$MNAR = \max_i |r_i| / \sigma_{r,i} \qquad (28)$$

Because of the maximization, the distribution of the *MNAR* is mathematically intractable. Yet, it is clear that the *MNAR* will grow with the value of μ_i and can be used to distinguish between the hypotheses just like the *SSE* in (27). Unlike the *SSE*-based technique discussed above, the element for which the absolute residual is largest indicates immediately which of the corrections is most likely to have been in failure.

The RSIM standard assumes a separate hypothesis test for each individual correction/residual as it states: “An alarm condition occurs if the absolute value of the PR residual exceeds the threshold…” However, as can be seen from the specification of RSIM message 20, only a single correction can be flagged as ‘failing’ simultaneously. Although this is not affirmed by the standard, it has been assumed here that in the case that multiple residuals simultaneously grow beyond the threshold, the PRN of the correction that is most likely to have failed is the one that is sent to the RS for further action. Not surprisingly, this is the correction that corresponds to the largest of those residual [Kelly97]. It is thus assumed that the test of interest uses the following test statistic:

$$MAR = \max_i |r_i| \qquad (29)$$

A failure is detected as soon as $MAR > MAR_{threshold}$ and is contributed to the correction that corresponds to the largest absolute residual. The algorithm that has been described in the RSIM standard is therefore strikingly similar to the *MNAR* algorithm. There is, however, an important difference: the failure detection test in the RSIM standard does not normalise the residual to unit standard deviation, as both the *SSE* and *MNAR* algorithms do (by dividing the residual by its standard deviation). This means there is only a single threshold, set by RSIM message 16, that is used for detection of biases in residuals with different noise levels, which complicates analysis.

5. RSIM PERFORMANCE ANALYSIS

As has been described in section 4.3, in case of a detected failure, the satellite that is excluded from the solution by means of an RSIM message 20 is the one of which the residual exceeds a certain threshold $MAR_{threshold}$ *and* the residuals from other satellites.

There are four possible system states that can occur. The probability that the system is in each of these states characterizes the performance:

- nominal: there is no failure and no PRN is flagged
- false alarms: there is no failure, yet a PRN is flagged
- missed detections: no failure is detected while one of the residuals is biased
- wrong identifications: a failure is detected but the wrong PRN is flagged

In the presence of an error, tuning should be such that the probability of missed detection is small, which requires a small threshold. Simultaneously, the probability of wrong identification should be small, which is promoted by a large threshold. As a side effect of failure-detection, in case there is no error, satellites can be identified as erroneous due to noise, which is called ‘false detection’. This degrades system continuity and its occurrence should be kept to a minimum, which requires a large threshold as well. The setting of the threshold thus balances the amount of false detections and wrong identifications with the number of missed detections. The distribution of the residuals depends on a number of parameters that are time varying and not available to the system operator, such as the standard deviations of the residual, the weights used in the clock bias estimations, and the magnitude of the failure induced bias. To set a threshold using RSIM message 16, it will therefore be unavoidable to make assumptions on these parameters. In the following, it is assumed that the threshold has been set to a value *T*, which could for example based on the assumptions of all equal weights and set such, that the probability of false alarms remains below a given limit.

The main goal of the remaining section is to show how the probabilities of false alarm, missed detection and wrong identification can be determined as a function of both *T* and the failure induced bias. The main 'tool' is the use of inequalities to reduce the integration of the N_{IM} dimensional distribution of the residual vector to the integration of a one- or two-dimensional distribution only. In all cases, every step should guarantee conservative performance results. For simplicity of notation, it will be assumed without loss of generality that the failure occurs in the correction indexed by *i*=1 and that the induced bias is positive, that is, $\mu_1 > 0$. Furthermore, it is assumed that the bias is known, that is, the performances are derived for a bias of a *given* size. Once the performance as a function of the bias is determined, one can go one step further and assess what happens when the bias size remains unknown.

5.1 False alarms

The probability of false alarms is relatively easy to assess. It is the probability that at least one of the residuals exceeds the threshold *in the absence of RS failure induced biases*:

$$P_{FA} = P(\max_i |r_i| > T \mid no\ bias) \tag{30}$$

Using the (first) Bonferroni inequality [Kelly96], one can find the following conservative upper bound on this false alarm probability:

$$P_{FA} \le \sum_{i=1}^{N} P(|r_i| > T \mid no\ bias) \tag{31}$$

Note that the problem of computing the probability is reduced to one dimension: the mutual distribution of all residuals is no longer needed: only the distributions of each of the separate residuals is required.

Another upper bound that could be used and that is somewhat tighter is the Joshi upper bound [Joshi72] that is in fact the probability that would be obtained in the absence of correlation between the residuals. It is given by

$$P_{FA} \le 1 - \prod_{i=1}^{N} P(|r_i| < T \mid no\ bias) \tag{32}$$

Again, the probability can be found from the distributions of the separate scalar residuals rather than a multidimensional mutual distribution.

5.2 Missed detections

The probability of missed detection is the probability that, while there is a bias in satellite 1, none of the residuals exceeds the threshold:

$$P_{MD}(\mu_1) = P(\max_i |r_i| < T \mid \mu_1) \tag{33}$$

A conservative upper bound to (33) can be obtained by replacing the maximum residual by a residual with a fixed index *k*. Because the k^{th} residual is always smaller or equal to the maximum residual (in an absolute sense), the following holds:

$$P_{MD}(\mu_1) \le P(|r_k| < T \mid \mu_1) \tag{34}$$

Any of the residuals could be used and leads to an upper bound. However, the residual that contains the largest bias will be the one that corresponds to the biased correction and will give the tightest bound. By assumption, this is the first correction (or residual) and therefore it is proposed to use the bound:

$$P_{MD}(\mu_1) \le P(|r_1| < T \mid \mu_1) \tag{35}$$

Again, the bound reduces the need for use of a multidimensional distribution to a scalar one.

5.3 Misidentification and wrong exclusion

The misidentification case is the most complicated one. When $P_{ID,i}$ denotes the probability that the i^{th} correction is identified as being in failure, the probability of misidentification reads:

$$P_{MID} = \sum_{i=2}^{N_{IM}} P_{ID,i} \tag{36}$$

The probability $P_{ID,i}$ can be written as:

$$P_{ID,i} = P(\forall j \ne i : |\underline{r}_i| > |\underline{r}_j| \wedge |\underline{r}_i| > T) \tag{37}$$

which shows that in order to identify the ith residual of being in failure two conditions need to be fulfilled simultaneously:

- The i^{th} residual is larger than all other residuals
- The i^{th} residual is larger than the treshold T

By dropping one of the conditions, the restrictions on the residual become less and the probability (37) becomes larger. The easiest way to proceed is to drop the first condition that the residual should be the largest and just consider the fact that the wrong residual exceeds the failure detection threshold:

$$P_{ID,i} \le P(|\underline{r}_i| > T) \tag{38}$$

In [Ober01], another approach, more complicated approach is followed that begins by dropping the second condition:

$$P_{ID,i} \leq P(\forall j \neq i : |\underline{r}_i| > |\underline{r}_j|) \quad (39)$$

This approach is especially suitable when large biases are introduced that make the probability that the threshold is exceeded close to one anyway, which implies that dropping the second condition will hardly make a difference. In that case, it will provide a tighter upper bound than the first approach described above. To bring the computational effort of (39) down, the union bound can be used to write the problem in terms of a two-dimensional distribution. For further details, the reader is referred to [Ober01].

6. USER PERFORMANCE

With the expressions of the previous section, it is possible to assess the performance a user obtains from the DGNSS service in a conservative manner. First note that the previous section has assumed knowledge on the bias, while in reality, it is hard to say anything about the bias. The biases that are of interest are not very small (as they wouldn't affect operations significantly) nor vary large (as very large biases are easy to detect and isolate).

The following procedure can be used to determine thresholds and the related performance. The maximum allowed false alarm probability $P_{FA,max}$ per independent sample is used to set the threshold T for error detection. Given T and a bias of a given size B it is possible to use the techniques discussed above to determine:

- the probability of missed detection $P_{MD}(B)$, that relates to the case when the bias remains completely undetected
- the probability of missed identification $P_{MI}(B)$, that relates to the case that a wrong satellite is indicated as erroneous

Using the expressions for $P_{MD}(B)$ and $P_{MD}(B)$, a minimum bias size can be determined that can be detected (or identified) with sufficiently high probability: the MDB or minimal detectable bias [Leva96], and its counterpart, the MDI or Minimal Identifiable Bias. It should be assumed that biases smaller than the MDB/MDI cannot be detected or identified sufficiently well and they can enter the position of the user of the service. The frequency with which biases occur (that should somehow be conservatively estimated or be determined by extensive measurement campaigns) determines the frequency the user performance is affected by biases in the corrections that the service broadcasts with the size of at most the MDB/MDI. In practice, the distinction between MDB and MDI might be dropped by taking only the largest and therefore most conservative of these two values.

7. CONCLUDING REMARKS

The paper has discussed the main integrity monitoring mechanism as implied by he RSIM standard. It has provided the tools to obtain a conservative assessment of the most important performance parameters. The main RSIM algorithm uses pseudorange residuals with different noise variances for failure detection, but has to compare them all to the same threshold, which is set by RSIM message 16. This not only prevents the use of standard algorithm performance assessment techniques from navigation literature as provided in [Kelly96,Kelly97], but will lead to sub-optimal performance as well. The author therefore expresses his hope that future version of the RSIM standard will take this into account and allow for normalized residuals.

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