LAAS Integrity the Bayesian Way

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BIOGRAPHY

Bastiaan Ober's areas of experience include the influence of multipath on GPS positioning, carrier phase differential GPS, ambiguity resolution and integrity monitoring, especially for aviation applications. He is currently working as a Ph.D. student doing research on integrity design and analysis of integrated navigation systems for safety critical applications.

ABSTRACT

Applying Bayesian analysis techniques, the paper shows that the position distribution can be derived from the measurement data as a weighed sum of error distributions, in which the weights depend both on the measurement residuals, the statistical properties of the measurements and the a priori probabilities of failure. This description is not only elegant in the insight it provides on the 'distribution' of integrity over the different measurements, but also offers major operational benefits over existing methods. Unlike with currently available methods, the *exact* probability of hazardously misleading information *P(HMI) is* readily derived, while the processing scheme is simpler than for traditional fault detection based architectures. Secondly, the method makes it particularly easy to identify the exact contribution of each individual measurement to *P(HMI)*, which simplifies the identification and removal of erroneous measurements. Finally, it becomes viable to do fault-tolerant position estimation in a way similar to the one described in [12] by a straight-forward minimization of *P(HMI)*.

1. INTRODUCTION

Integrity and continuity requirements for aircraft precision approach and landing are extremely stringent. In such an operational environment, it is important to exploit all available information to the fullest possible extent to assess system integrity as accurately as possible - to avoid both being optimistic (compromising integrity) and conservative (sacrificing continuity).

To this end, a new method has been developed for failure mitigation. It allows for a unified treatment of the system both in the absence and presence of failures, in contrast to the situation for currently available fault detection based algorithms. This simplifies computations and gives a more accurate assessment of system integrity.

The paper is organized in the following manner. After a brief introduction on navigation performance requirements and a description of the system model used, a short overview of currently available fault detection and exclusion (FDE) algorithms is presented, with a stress on the difficulties of FDE to obtain accurate performance measures. It is then shown how Bayesian techniques can be exploited to overcome these problems, and how they can be applied in a Local Area Augmentation System (LAAS). Because the method resembles the multiple hypothesis approach from [12], results are compared to those described in that paper. It is thus shown that because the multiple hypothesis approach doesn't exploit the fact that the measurements contain information on the likelihood of a possible failure, it may lead to overly optimistic results.

2. PARAMETER ESTIMATION PERFORMANCE

Due to the random behavior of most error sources in a GNSS system, the parameters it provides for navigation are random as well and can therefore best be described by a probability density function (pdf). For LAAS, the parameters of interest are the pseudorange corrections at the ground station, and the position at the aircraft.

It has become a custom to capture the most important characteristics of the pdf in two different values. The first parameter is *accuracy* and describes the 'bulk performance' that is usually dominated by small, noisy error sources that are always present in the system. It is typically defined at the 95% level. The second parameter is the *protection level* and represents the worst case performance that is typically dominated by the presence of failures. It is defined at a level of 99.9…9% where the actual number of nines depends on the type of operation that is performed.

When system failures cause the protection limit to degrade beyond the maximum allowed level called *alarm limit*, this makes the systems effectively unusable. However, the effect of failures can be mitigated by failure detection (FD). In the case of a detected error, the use of the system can be prevented. The worst case performance can then be considered conditioned on the absence of detected failures which typically improves the protection level considerably. The probability that the protection level exceeds the alarm level in the absence of a timely notification of the user is called the *integrity* of the system. In case of a notification, the system can no longer be used for the intended operation. The probability that this occurs is called the system's *continuity of function*.

To improve the continuity of service of a system, one can go one step further than mere failure detection. When the failure can be attributed to one particular satellite, this satellite can be excluded from the parameter estimation. This is called Fault Detection and Exclusion (FDE). Both detection and exclusion are based on a measure of the measurement errors, usually in the form of a (least squares) residual [2,4,14].

Presence of sufficient accuracy, integrity and continuity of function is required to make a system available for a certain operation. Therefore, the system performance needs to be *monitored*. In many circumstances, it is also beneficial (or required) to *predict* system performance in advance. Where performance monitoring can exploit both measurement models and the actual signals, prediction necessarily relies on models only.

3. THE SYSTEM MODEL

In this paper, the system is assumed to be sufficiently well described by an overdetermined ($N>M$) set of linear equations that relate the measurements to the unknown parameters as

$$\underline{z} = H\underline{x} + \underline{n} + \underline{b} \tag{1}$$

with

$\underline{z}$: N-vector of measurements
H: $N \times M$ observation matrix
$\underline{x}$: M-vector with unknown parameters
$\underline{n}$: N-vector of measurement noise
$\underline{b}$: N-vector of measurement biases

When a random D-vector $\underline{d}$ has a normal distribution with mean $\underline{\mu}_d$ and covariance matrix Σ_d this is written as $\underline{d} \sim N(\underline{\mu}_d, \Sigma_d)$. Although this is not standard notation, $N_{\tilde{d}}(\underline{\mu}_d, \Sigma_d)$ will be used as a shorthand for the probability $P(\underline{d} = \tilde{\underline{d}})$ that $\underline{d}$ has a specific value $\tilde{\underline{d}}$:

$$N_{\tilde{d}}(\underline{\mu}_d, \Sigma_d) = [(2\pi)^D \det(\Sigma_d)]^{-\frac{1}{2}} e^{-\frac{1}{2}(\tilde{\underline{d}} - \underline{\mu}_d)^T \Sigma_d^{-1} (\tilde{\underline{d}} - \underline{\mu}_d)}$$

It is assumed that the measurement noise is normally distributed with zero mean and covariance-matrix Σ_n:

$$\underline{n} \sim N(\underline{0}, \Sigma_n) \tag{2}$$

For simplicity it will be presumed that the simultaneous occurrence of multiple failures is too rare to require full investigation. This is justifiable when it is conservatively assumed that all these cases automatically lead to excessive estimation errors. As only the cases of zero or one failure are considered, it is convenient to use E_i to denote the event of a failure in measurement i and represent the no failure case by E_0. The a priori probability of occurrence of each event is denoted by $P(E_i)$.

Under the no failure case E_0, the measurements are assumed unbiased, while under E_i the failing measurement contains a bias of unknown size B_i:

$$\underline{b} = \begin{cases} \underline{0} & under\ E_0 \\ B_i \underline{e}_i & under\ E_i,\ i > 0 \end{cases} \tag{3}$$

with $\underline{e}_i = [\underbrace{0 \cdots 0}_{1 \ldots i-1} \ \underbrace{1}_{i} \ \underbrace{0 \cdots 0}_{i+1 \ldots N}]^T$. Therefore, (1) in fact represents several different models that apply to different (failure) modes of the system. It is this model that is generally used for FDE based architectures.

For the Bayesian analysis, a different model will be used. When measurements can contain failures they can be modeled by a mix of probability density functions, representing the nominal and failure cases respectively, weighed by their respective probabilities of occurrence:

$$P(\underline{z} = \tilde{\underline{z}}) = \sum_{i=0}^{N} P(E_i) N_{\tilde{z}}(B_i \underline{e}_i, \Sigma_n) \tag{4}$$

Note that this a rather different starting point than (1), that in fact – in combination with (3) - represents different models for the 'failure' and the 'no failure' case. On the other hand, (4) incorporates all cases and their probability of occurrence simultaneously.

4. FDE BASED ESTIMATION SCHEMES

A general architecture of an FDE based parameter estimation scheme is depicted in Figure 1. Two monitoring functions monitor accuracy and integrity of the estimated parameters. The integrity monitor determines whether there is sufficient redundancy to detect failures with a sufficiently high probability. When this is not the case, it provides a warning (yellow light) that the integrity of the system can not be guaranteed. When a fault is detected, an iterative procedure determines whether the fault condition disappears when the measurement set is reduced. A detection only results in a red light when this is not the case. Of course, the reduced measurement set might not possess sufficient error detection power anymore, which would result in an integrity warning.

Both the ground and airborne processing schemes of LAAS architectures as the ones in [8,12] are represented by Figure 1. For the ground based system part, a red sign would lead to the blockage of use of a GNSS pseudorange correction, while the yellow light is generally not used.

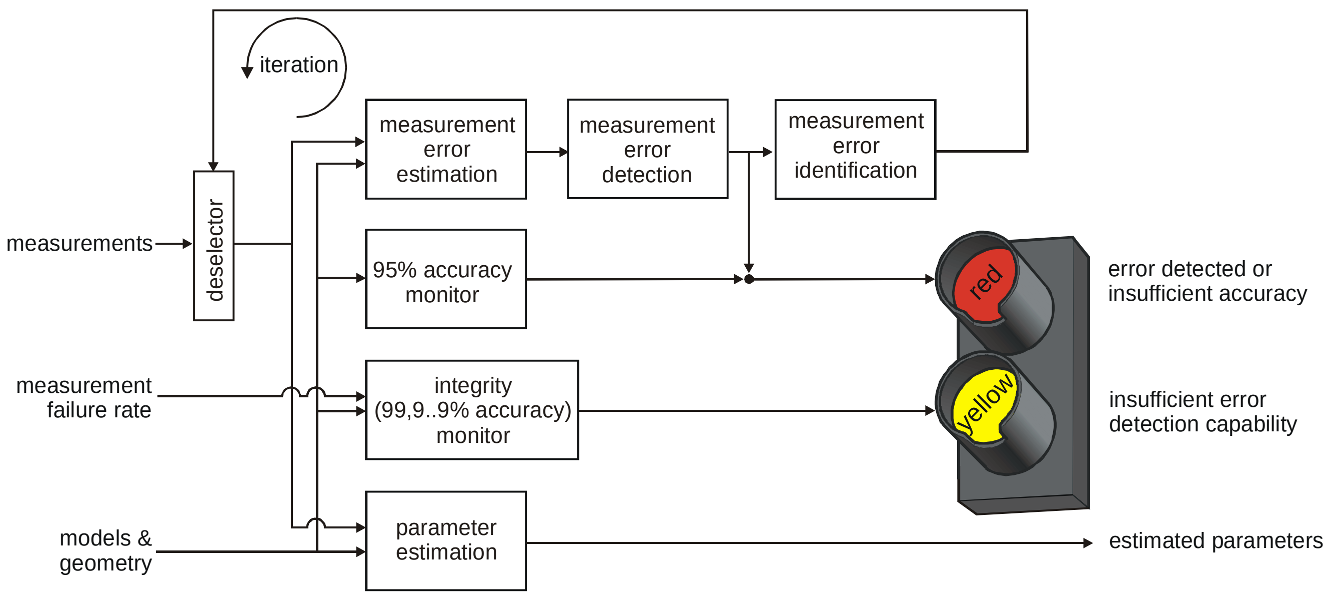

Figure 1. *The architecture of an FDE based GNSS system*

4.1 FDE Performance

To describe the performance of an FDE based architecture, it is useful to introduce the following notational conventions:

$\hat{\underline{x}}$: the estimated parameters
X: set of all estimation errors within the alarm limit
T: test statistic used for error detection
h: threshold for error detection

When the real parameter values are written as $\underline{x}$, requirements are given in terms of the parameter deviation, defined as:

$$\Delta\hat{\underline{x}} = \underline{x} - \hat{\underline{x}} \tag{5}$$

Using the symbols above, the probability of a *estimation failure*, when the parameter deviation exceeds acceptable bounds, is $P(\Delta\hat{\underline{x}} \notin X)$, while the probability of a detected failure equals $P(T > h)$. It has been proven in [9] that these probabilities are statistically independent for test statistics based on the (least squares) residual. Therefore, the probability of an undetected estimation failure is

$$P(HMI) = P(\Delta\hat{\underline{x}} \notin X)P(T < h) \tag{6}$$

where HMI stands for Hazardously Misleading Information. Because system behavior will be substantially different under the different events E_i, it is convenient to split *P(HMI)*. Writing it explicitly as the sum of the probabilities of HMI under each of the events E_i it reads:

$$P(HMI) = \sum_{i=0}^{N} P(HMI \mid E_i) \tag{7}$$

in which

$$P(HMI \mid E_i) = P(E_i)P(\Delta\hat{\underline{x}} \notin X \mid E_i)P(T < h \mid E_i) \tag{8}$$

In case of failure, both $P(\Delta\hat{\underline{x}} \notin X \mid E_i)$ and $P(T < h \mid E_i)$ depend on the unknown bias B_i, which prevents their computation. There are two 'solutions' to solve this problem, that are discussed in more detail in [10].

The first solution is to replace the bias by the minimal detectable bias (MDB) [7,14], which is the smallest satellite bias that can be detected with a probability of at least a given probability α. However, biases smaller than the MDB have a reduced detectability but can still cause estimation failures. For certain combinations of satellite geometry and satellite errors, the use of the MDB can therefore lead to underestimation of the probability of HMI.

The possibility that the MDB approach could be underestimating the missed detection probability for certain satellite biases has been realized for some years now. Therefore, [3,5,9] all propose to substitute a worst case bias, that maximizes *P(HMI|E$_i$)*. Consequently, worst case bias substitution never underestimates this probability, but is computationally rather involved.

In both cases, the unknown bias is substituted by a bias of a particular size that depends only on the system models and not on the received measurements. These substitution methods are therefore usable for off-line computations and

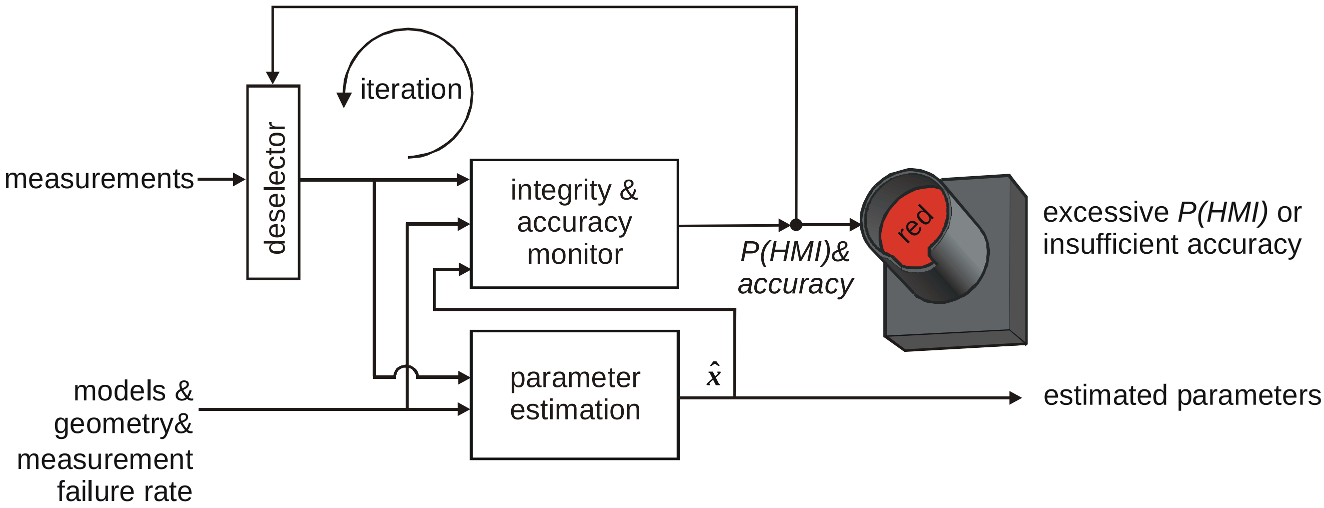

Figure 2. *The simplified architecture of a system using Bayesian analysis*

can be applied to both performance monitoring and prediction. The exclusion part of FDE will not be addressed separately here. Suffice it to say that the presence of the unknown bias poses similar problems as in the detection part.

4.2 Disadvantageous of FDE

FDE has a number of drawbacks that mainly arise from the problem of having to deal with an unknown bias. Summarizing, the following problems have been touched upon in the previous section:

- The unknown bias problem leads to computational complexities and inaccurate *P(HMI)* values and therefore either reduces continuity of service and unavailability or to underestimation of *P(HMI)*.
- For performance monitoring purposes, the information of the bias is not optimally used as it is only reflected in the decision between ‘failure detected’ and ‘no failure detected’.
- The use of separate estimation, fault detection and fault exclusion algorithms leads to a complex system architecture.

The next section will introduce a new, simplified processing scheme for performance monitoring that does not have any of the shortcomings listed above.

5. A NEW BAYESIAN APPROACH

In this section, the traditional separation between the cases of absence and presence of failures will be replaced by an integrated Bayesian approach that uses a priori weights to indicate the probability of the presence of failures. While in FDE environments these failure probabilities are only used to derive the requirements on the failure detection algorithm, they are an integral part of the analysis presented here. Note that from now on, the mixture model (4) is applied instead of (1).

A commonly used tool in (Bayesian) statistical interference [1] is the likelihood function, that is the probability density of the measurements *regarded as a function of the unknown parameters*:

$$L(\underset{\sim}{\tilde{x}}) = P(\underset{\sim}{\tilde{z}} \mid \underline{x} = \underset{\sim}{\tilde{x}}) \qquad (9)$$

$L(\underset{\sim}{\tilde{x}})$ measures the relative likelihood that a particular value $\underset{\sim}{\tilde{x}}$ of the unknowns has given rise to the observed value of the measurements $\underset{\sim}{\tilde{z}}$. Given the form of the measurements’ distribution (4), the likelihood can be written as

$$L(\underset{\sim}{\tilde{x}}) = \sum_{i=0}^{N} P(E_i) L(\underset{\sim}{\tilde{x}}, E_i) \qquad (10)$$

in which the likelihood under each of the events E_i is

$$L(\underset{\sim}{\tilde{x}}, E_i) = P(\underset{\sim}{\tilde{z}} \mid \underline{x} = \underset{\sim}{\tilde{x}}, E_i) \qquad (11)$$

When no a priori distribution information about the unknowns is present, Bayes’ rule gives their (a posteriori) distribution given the measurements as [6]

$$P(\underline{x} = \underset{\sim}{\tilde{x}}) = C \cdot L(\underset{\sim}{\tilde{x}}) = C \cdot \sum_{i=0}^{N} P(E_i) L(\underset{\sim}{\tilde{x}}, E_i) \qquad (12)$$

where C is a constant that ensures that (12) represents a distribution and integrates to one. Appendix A shows that the likelihood has the form:

$$L(\underset{\sim}{\tilde{x}} \mid E_i) = c_i \cdot N_{\underset{\sim}{\tilde{x}}}(H_{(i)}^{+} \underline{z}_{(i)}, \Sigma_{x,i}) \qquad (13)$$

Substitution in (12) and determining C such that it integrates to one gives the following mixture description for the probability that the $\underline{x}$ equals $\underset{\sim}{\tilde{x}}$:

$$P(\underline{x}=\underline{\tilde{x}}\,|\,\underline{\tilde{z}})=\sum_{i=0}^{N} w_i N_{\tilde{x}}(H^{+}_{(i)}\underline{\tilde{z}}_{(i)},\Sigma_{x,i}) \quad (14)$$

with the weights of the distributions belonging to the events E_i equal to:

$$w_i = P(E_i)c_i \cdot [\sum_{k=0}^{N} P(E_k)c_k]^{-1} \quad (15)$$

Note that expression (15) is similar to the one in [12] in which the weights are set to $P(E_i)$ and therefore don't reflect any information from the measurements. Here, however, one expression contains *all* information on the parameters to be estimated *and* the possible presence of biases, as the weights represent the a posteriori probabilities of the events E_i to have occurred.

Because one equation contains all relevant information for estimation, fault detection and fault exclusion, a system architecture based on the use of the analysis presented here can be much simpler, see Figure 2, than was the case for the FDE based system of Figure 1.

Just like one would expect, the weights for E_i become larger when:

- the a priori probability $P(E_i)$ is higher
- the residuals under E_i are smaller
- the accuracy under E_i is better
- the non-failing measurements under E_i have smaller covariance

An example of two realizations of (14) is given in Figure 3 for the E_0 and the E_1 case.

For an *arbitrary* parameter estimate $\hat{\underline{x}}$ the probability of an estimation failure is readily derived from (12) as:

$$\int_{\underline{\tilde{x}}-\hat{\underline{x}}\notin X} P(\underline{x}=\underline{\tilde{x}}\,|\,\underline{\tilde{z}})d\underline{\tilde{x}} = \sum_{i=0}^{N} w_i \int_{\underline{\tilde{x}}\notin X} N_{\tilde{x}}(H^{+}_{(i)}\underline{\tilde{z}}_{(i)}-\hat{\underline{x}},\Sigma_{x,i})d\underline{\tilde{x}} \quad (16)$$

When a warning is issued to the user whenever (16) exceeds the allowed maximum probability of *P(HMI)*, the probability of an estimation failure without warning automatically becomes equal to the probability of an

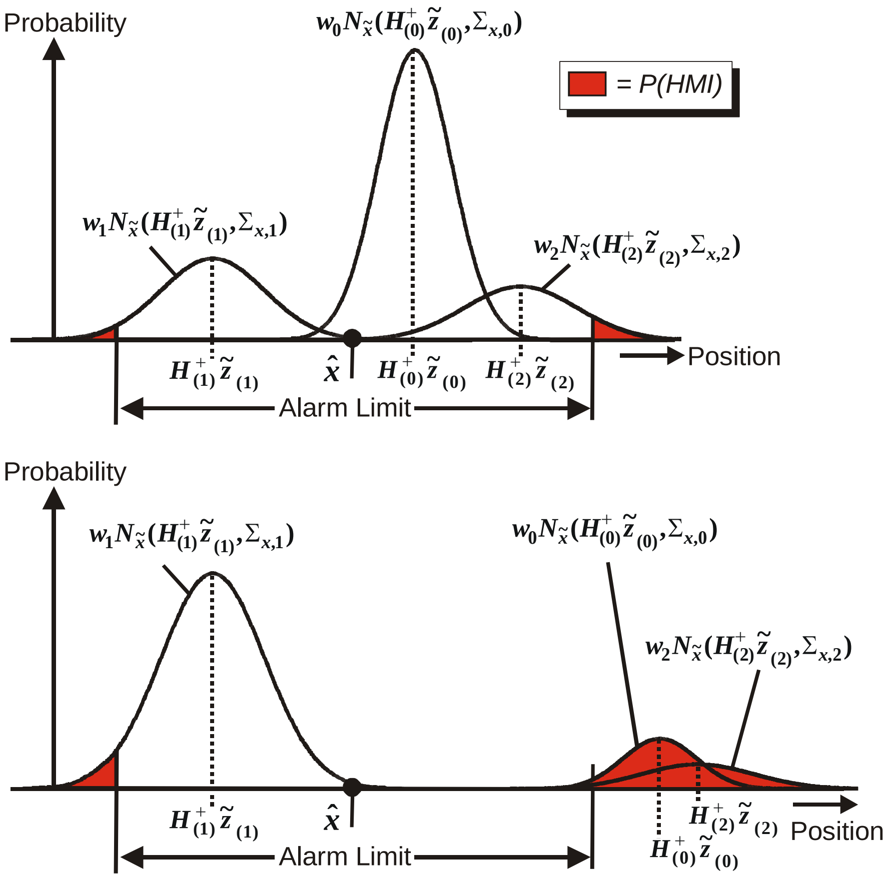

Figure 3. *The mixture distribution in the case of no failures and one failure respectively. In the first case, the mixture is dominated by the 'no failures' distribution, in the second case, by the 'failure in measurement 1' distribution. The alarm limit and P(HMI) are indicated for an arbitrary parameter estimate* $\hat{x}$.

estimation failure (16) itself! Unlike the case for FDE, the answer is exact, as it does not depend on the unknown value of the failure-induced bias. For the 95% accuracy, a similar approach can be followed.

For exclusion purposes, the weights in (14) are very useful as they indicate the a posteriori probabilities that a failure has occurred in a particular measurement. Therefore, the measurement that corresponds to the event with the largest weight is the 'maximum likelihood' choice for exclusion.

5.1 High Integrity Parameter estimation

As [12] has shown, the simple form that *P(HMI)* takes can be exploited to estimate the parameters of interest in such a way, that *P(HMI)* is minimized. Because this effectively desensitizes the estimate for measurement biases and thereby improves integrity, this is a form of High Integrity Parameter (HIP) estimation [11], which can improve system availability.

6. APPLICATION TO LAAS

In this section, the use of the Bayesian analysis is discussed more specifically for a LAAS setup. After a short indication of possible uses for the ground processing function, a slightly more detailed account is given of its use in the aircraft. In particular, because of the similarity of (14) with the expressions found in [12], the two methods are compared by a small simulation.

6.1 Ground processing

The Bayesian method can be used for different purposes, depending on the way the LAAS is setup. In LAAS architectures such as the one proposed in [8], the ground processing function performs fault detection and exclusion based on a continuity-based threshold before it sends out pseudorange corrections. In this situation, the FDE function might be replaced by exploitation of (14) with all the advantageous earlier discussed. However, it will be necessary to base the system on an alarm limit for the pseudorange corrections.

When the ground station does not do any FDE but provides the aircraft with both uncensored corrections and correction quality data such as in [12], one of the things that could be considered would be to use the mixture distribution parameters for quality assessment. This would allow for derivation of a mixture distribution for the aircraft position, and therefore for exact computation of its horizontal and vertical protection level. Another possibility would be to use the protection level of the corrections for this purpose.

6.2 Airborne function

The airborne function of a LAAS architecture will always use some kind of failure detection on its position computations. Instead of FDE, this process might be based on the Bayesian method developed here, possibly in combination with the same method on the ground as discussed above.

Because the results from the Bayesian method resemble the multiple hypothesis approach from [12], it is interesting to do some comparisons with the results from that paper. The important difference between the multiple hypothesis and the Bayesian method is, that instead of (15) the former uses weights equal to the a priori probability of the events *Ei*:

$$w_i = P(E_i) \tag{17}$$

These weights don't reflect the fact that the measurements contain information on the likelihood of a possible failure.

Although a full comparison is beyond the scope of this paper, some results will be given in this section. All system models and parameters are chosen to be consistent with [12]. Only a small amount of data has been processed. Therefore, the actual values of the parameters and the corresponding performance figures are less important than the observed trends.

The model is based on the processing of 3 different sets of measurements from the ground station to provide three different estimates z_i of the vertical position x_v:

$$\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} x_v + \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} n_{g1} \\ n_{g2} \\ n_{g3} \\ n_a \end{bmatrix}$$

where the airborne noise is given as

$$n_a \sim N(0, \sigma_a^2)$$

and the ground related noise components (including geometry effects) equal

$$n_{gi} \sim \begin{cases} N(0, \sigma_g^2) & under\ E_0 \\ N(B_i, \sigma_g^2) & under\ E_i, i > 0 \end{cases}$$

The a priori probability that the z_i contain corrections that contain data from a failing receiver at the ground station is given by

$$P(E_i) = \begin{cases} 1 - 3 \cdot 10^{-5} & for\ i = 0 \\ 10^{-5} & for\ i = 1,2,3 \end{cases}$$

Although [12] provides no values for the standard deviations, it can be derived from the examples that

$$\sigma_a \approx 0.23\text{m}$$
$$\sigma_g \approx 4.7 \cdot \sigma_a$$

Using these parameter values, two different ways to compute *P(HMI)* have been investigated:

- The Bayesian approach, using weights from (15). The result is referred to as $P_B(HMI)$
- The multiple hypothesis approach (MHA) using (17). The result is referred to as $P_{MHA}(HMI)$

Three different estimators are computed:

- Least Squares, given by $H^+\underline{z}$
- HIP estimator obtained by minimization of *P(HMI)* as obtained by the MHA, referred to as the *MHA estimator*
- HIP estimator obtained by minimization of *P(HMI)* as obtained by the Bayesian approach, referred to as the *Bayes' estimator*

Three different failure modes were considered:

- No failure
- One failure in measurement 1 with a bias of B_1=VAL
- One failure in measurement 1 with a bias of B_1=5VAL

Some results for a modest 30 samples of data are presented in Figure 4 and 5.

Figure 4 shows that $P_{MHA}(HMI)$ for zero or small biases is optimistic for the least squares and MHA estimators, but conservative for the Bayes' HIP estimator. For large biases, the least squares solution obviously becomes useless, but so does the MHA estimator. Furthermore, $P_{MHA}(HMI)$ becomes an unreliable measure of the probability of *HMI*.

This can be explained as follows: for Bayesian analysis, in the presence of a large bias, all weights in the mixture become zero except the one that corresponds to the bias-

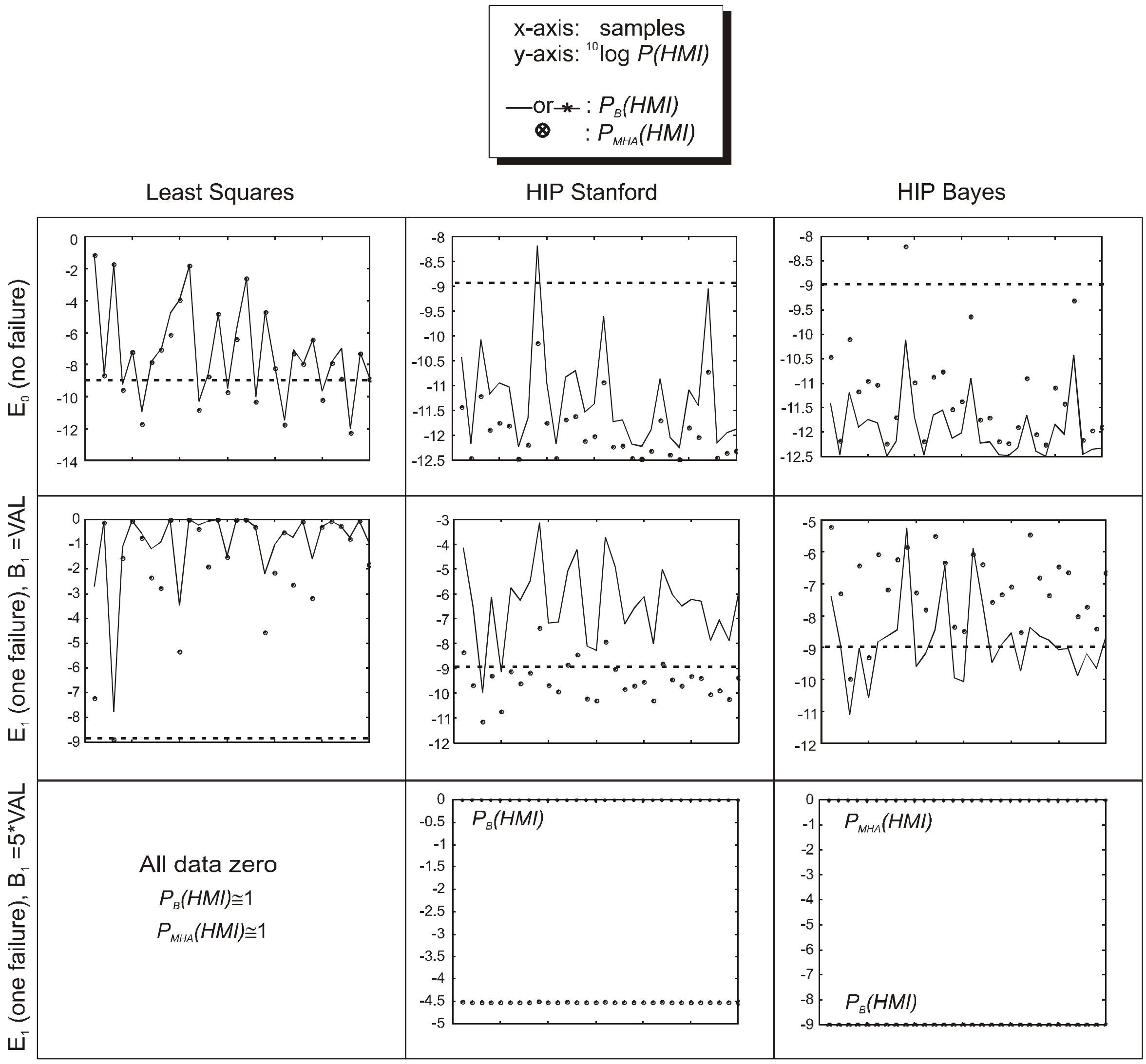

Figure 4. *The probability of HMI for three different estimators under conditions of no failure and one failure of size VAL and 5VAL assessed by both the Stanford method of [12] and the Bayesian method developed in this paper*

free estimator the estimator, that therefore becomes equal to $H^{+}_{(i)}\underline{z}$, while the covariance is constant. For the MHA case, the biased and unbiased components in the mixture are driven apart. Because the weights are constant, this means that the MHA estimator will be drawn towards the distribution with the heighest weight, which is the one under E_0. It therefore behaves similar to the least squares estimate and is not really robust against failures, as a result of the fact that, although the data clearly shows that there is a failure in measurement 1, this is not accounted for in the weighing of the distributions.

Figure 4 also shows that HIP estimation can improve *P(HMI)* by several orders of magnitude. It is however still to be verified that this also leads to increased system availability.

Figure 5 compares the performance of the MHA estimator measured with $P_{MHA}(HMI)$ with that of the Bayesian estimator measured with $P_B(HMI)$. In the absence of failures or in the presence of small biases, the MHA seems to be consistently optimistic. For large biases however, it is conservative in its assessment of the lowest *P(HMI)* that can be achieved for a given sample. The estimator it provides, however, is way out of bound!

7. CONCLUDING REMARKS

A Bayesian method has been developed, that exploits both *all* a priori knowledge (in the form of a number of system parameters) and *all* measurement data to derive *exact* integrity information. It therefore provides a powerful tool for system performance analysis. A further advantage of the method is a simplified computational scheme that is more transparent than the one used with traditional fault detection and exclusion (FDE) based architectures. All monitoring and FDE related functionality is now performed by one integrity and accuracy monitor based on equation (14). As such, it can be used wherever FDE is used, and is not limited to LAAS.

The results of the Bayesian method have been compared to those obtained in [12] by the multiple hypothesis approach for a typical LAAS configuration. It seems that the multiple hypothesis approach sometimes gives too optimistic an assessment of the actual probability of hazardously misleading information that is achieved.

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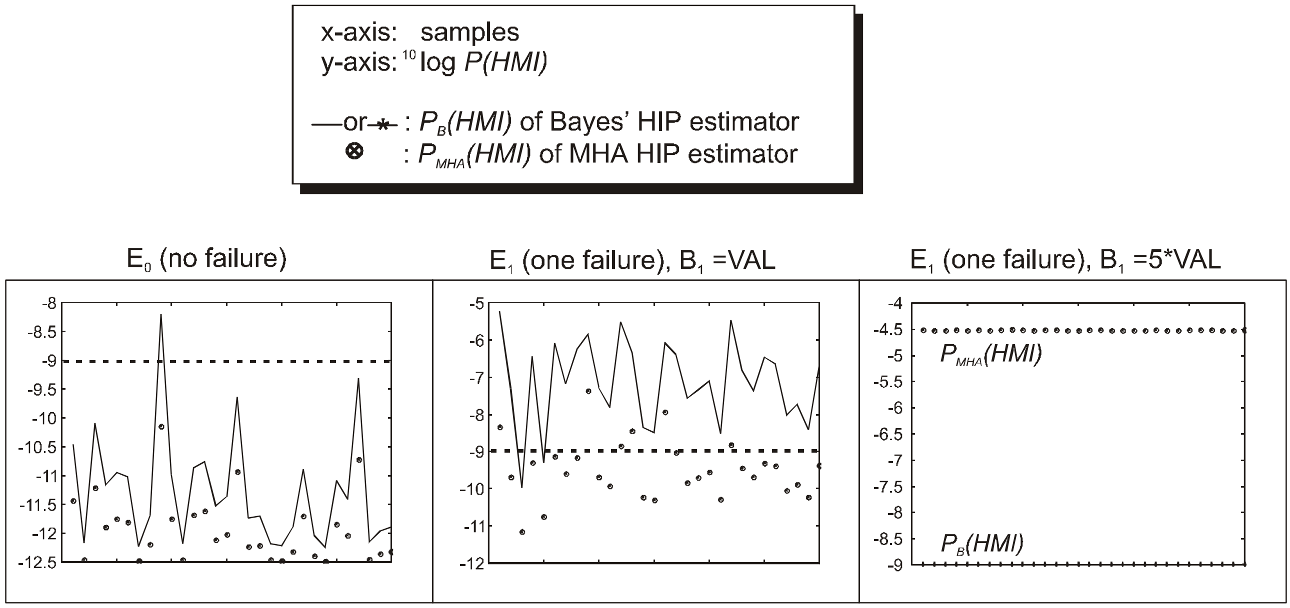

Figure 5. *The minimum probability of HMI that can be obtained for a certain sample as assessed by the multiple hypothesis approach from [12] and the Bayesian method developed in this paper, under conditions of no failure, one failure of size VAL and one failure of size 5VAL*

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APPENDIX A

In the no failure case ($i=0$), the likelihood is readily found from the measurement distributions as:

$$\begin{aligned} L(\tilde{\underline{x}} \mid E_0) &= k_0 e^{-\frac{1}{2}(\tilde{\underline{z}}-H\tilde{\underline{x}})^T \Sigma_n^{-1}(\tilde{\underline{z}}-H\tilde{\underline{x}})} \\ &= k_0 e^{-\frac{1}{2}S_o^2-\frac{1}{2}(\tilde{\underline{x}}-H^+\tilde{\underline{z}})^T \Sigma_{x0}^{-1}(\tilde{\underline{x}}-H^+\tilde{\underline{z}})} \end{aligned} \qquad \text{(A.1)}$$

with k_0 , Σ_{x0} and H^+ defined by

$$\begin{aligned} k_0 &= [(2\pi)^N \det(\Sigma_n)]^{-\frac{1}{2}} \\ \Sigma_{x0} &= (H^T \Sigma_n^{-1} H)^{-1} \\ H^+ &= (H^T \Sigma_n^{-1} H)^{-1} H^T \Sigma_n^{-1} \end{aligned} \qquad \text{(A.2)}$$

and the sum of the squared residuals S_0^2 given by

$$S_0^2 = \tilde{\underline{z}}^T (I - HH^+)^T \Sigma_n^{-1} (I - HH^+) \tilde{\underline{z}} \qquad \text{(A.3)}$$

In the case of a failure ($i>0$), the measurement distribution depends on the unknown bias B_i. As the bias is only a nuisance parameter for the problem at hand, it can be integrated out to arrive at the marginal distribution of the measurements as a function of the unknowns again, and the likelihood thus takes the form

$$L(\tilde{\underline{x}} \mid E_i) = k_0 \int_{-\infty}^{\infty} e^{-\frac{1}{2}(\tilde{\underline{z}}-B_i\underline{e}_i-H\tilde{\underline{x}})^T \Sigma_n^{-1}(\tilde{\underline{z}}-B_i\underline{e}_i-H\tilde{\underline{x}})} dB_i \qquad \text{(A.4)}$$

As the bias influences only the i^{th} element of the measurements, the best way to proceed is to separate the i^{th} element from the others. To this end, the following notations are introduced:

$H_{(i)}$: H with the i^{th} row removed

$\underline{z}_{(i)}$: $\underline{z}$ with the i^{th} row removed

$\Sigma_{n(i)}$: Σ_n with the i^{th} row and column removed

σ_i^2 : conditional variance of $\underline{e}_i^T \underline{n}$ for given $\underline{n}_{(i)}$

to be able to write the integrand in (A.4) in the form:

$$e^{-\frac{1}{2}\sigma_i^{-2}(a-B_i)^2-\frac{1}{2}(\tilde{\underline{z}}_{(i)}-H_{(i)}\tilde{\underline{x}})^T \Sigma_{n(i)}^{-1}(\tilde{\underline{z}}_{(i)}-H_{(i)}\tilde{\underline{x}})} \qquad \text{(A.5)}$$

The bias is now readily integrated out. Because for all real a it is known that

$$\int_{-\infty}^{\infty} e^{-\frac{1}{2}\sigma_i^{-2}(a-B_i)^2} dB_i = \sigma_i (2\pi)^{\frac{1}{2}} \qquad \text{(A.6)}$$

the likelihood can be written as:

$$\begin{aligned} L(\tilde{\underline{x}} \mid E_i) &= k_i e^{-\frac{1}{2}(\tilde{\underline{z}}_{(i)}-H_{(i)}\tilde{\underline{x}})^T \Sigma_{n(i)}^{-1}(\tilde{\underline{z}}_{(i)}-H_{(i)}\tilde{\underline{x}})} \\ &= k_i e^{-\frac{1}{2}S_{(i)}^2-\frac{1}{2}(\tilde{\underline{x}}-H_{(i)}^+\underline{z}_{(i)})^T \Sigma_{x(i)}^{-1}(\tilde{\underline{x}}-H_{(i)}^+\underline{z}_{(i)})} \end{aligned} \qquad \text{(A.7)}$$

in which $\Sigma_{x(i)}$, $H_{(i)}^+$ and $S_{(i)}^2$ are defined as above with H replaced by $H_{(i)}$, and

$$k_i = [(2\pi)^{N-1} \det(\Sigma_{n(i)})]^{\frac{1}{2}} \qquad \text{(A.8)}$$

When for the non failure case a consistent notation is defined by setting $H_{(0)} = H$, $\Sigma_{n(0)} = \Sigma_n$ etcetera, the likelihood under all events E_i can be written in the form

$$L(\tilde{\underline{x}} \mid E_i) = c_i \cdot N_{\tilde{x}}(H_{(i)}^+ \underline{z}_{(i)}, \Sigma_{x,i}) \qquad \text{(A.9)}$$

in which

$$\Sigma_{x,i} = (H_{(i)}^T \Sigma_{n(i)}^{-1} H_{(i)})^{-1} \qquad \text{(A.10)}$$

and

$$c_i = k_i e^{-S_{(i)}^2} [(2\pi)^M \det(\Sigma_{x,i})]^{\frac{1}{2}} \quad \text{(A.11)}$$