# RAIM PERFORMANCE: HOW ALGORITHMS DIFFER

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## BIOGRAPHY

Bastiaan Ober's areas of experience include the influence of multipath on GPS positioning, carrier phase differential GPS, ambiguity resolution and integrity monitoring. He is currently working as a Ph.D. student doing research on integrity monitoring algorithms for integrated navigation systems.

## ABSTRACT

Pilots that want to fly Basic Area Navigation in ECAC airspace using stand-alone GPS will be required to check the availability of RAIM for the intended flight (route and time) before taking off. One means of doing this is by using the RAIM availability prediction software AUGUR, that has been developed by STASYS Limited for EUROCONTROL. The design of the AUGUR algorithm has been done by Delft University.

The paper describes the considerations that have played a role in the selection of the RAIM prediction algorithm for AUGUR. One of the difficulties faced was to accommodate a wide range of different receivers with unknown RAIM implementations. The approach followed is to first categorise existing RAIM algorithms in a systematic way, to describe where they differ and what implications this has for their predicted performance. The resulting insights are then used to select an algorithm that obeys the requirements that are set by the JAA.

## 1. INTRODUCTION

From 23 April 1998 the carriage of equipment meeting the Basic Area Navigation (B-RNAV) became mandatory in all ECAC en-route airspace. The navigation requirements were based on the performance demonstrated by VOR/DME and multi-DME RNAV systems. However, many users saw stand-alone GPS as an economic means of compliance. Initially there was some doubt as to whether satellite navigation alone could provide sufficient integrity. Following a detailed review of aircraft systems, it was deemed by the Joint Aviation Authorities (JAA) that GPS can meet the B-RNAV requirements as long as the receivers are certified to TSO-C129 [TSO-C129] (henceforth referred to as TSO-C129 receivers) and obey some extra requirements laid down in [TGL-2]. Furthermore, GPS users will have to confirm during the pre-flight planning phase that the Receiver Autonomous Integrity Monitoring (RAIM) algorithm of their receiver will provide sufficient integrity for the intended flight (route and time), by using a prediction programme. Flight dispatch should not be made in the case where predicted continuous loss of RAIM of more than 5 minutes occurs for any part of the intended flight. A prediction programme, AUGUR has been developed by STASYS Limited for EUROCONTROL, and is made available through the World Wide Web.

This paper focuses on the JAA requirement that addresses the choice of the RAIM algorithm that is to be used in the prediction:

> *"The program should use either a RAIM algorithm identical to that used in the airborne equipment, or an algorithm based on assumptions for RAIM prediction which give a more conservative result".*

More information on the other requirements for AUGUR will be made available in [Harriman98] that focuses more on operational and implementation issues. AUGUR itself can be found at http://augur.ecacnav.com (Europe) or at the US mirror site http://augur.us.ecacnav.com.

AUGUR provides RAIM availability for both baro- and non-baro-aided GPS. Because the same algorithm is used for both cases, the sections on algorithm selection describe the non-baro-aided situation only.

## 2. ALGORITH SELECTION APPROACH

TSO-C129 certified receivers are required to perform Receiver Autonomous Integrity Monitoring (RAIM) to ensure that the majority of errors in GPS satellite signals can be detected before they cause large position errors. RAIM consists of two algorithms: an error detection algorithm, that detects actual satellite failures, and a 'geometry screening' algorithm, that determines whether the measurement geometry is sufficiently strong to provide the required missed detection probability and alarm rate. Geometry screening consists of computing the error detection performance as a function of the geometry and deciding whether this performance meets the requirements. As geometry screening does not depend on the actual GPS signals, its behaviour can be predicted for any user position and time instant. AUGUR does such prediction with a purely operational goal: it aims at highlighting whether user receivers are likely to experience any RAIM outages during the flight. In other words: AUGUR tries to predict the behaviour of the RAIM availability algorithm in TSO-C129 receivers rather than anything else.

The major problem in assessing receiver behaviour is that there are many different TSO-C129 receivers on the market. The RAIM algorithms used in these receivers are generally proprietary and thus unavailable in the public domain. In accordance with the JAA requirements, AUGUR must err on the safe side and give a conservative prediction when exact prediction is impossible. On the other hand, there is no need to sacrifice performance unnecessarily. The approach taken is therefore to be conservative within reasonable limits. These limits are set by the numerous algorithms and techniques that have been published in open literature. This paper assesses where conservative results can be expected from these techniques.

This paper first shows how RAIM performance can be computed *exactly* as a function of the measurement geometry. It then shows which approximations and simplifications can be applied to arrive at practical, existing RAIM algorithms from open literature. This structured approach reveals very distinctively in which respect(s) algorithms might differ, and which differences in performance can be expected. In this way, this paper can serve as a 'guide' to different RAIM algorithms, and enables a structured classification of algorithms by listing the specific approximations and simplifications used by each one.

Along the way, the paper justifies the choice of the geometry screening algorithm that has been implemented in EUROCONTROL's AUGUR RAIM prediction software. The JAA requires that the performance of the implemented algorithm should be conservative. The final algorithm therefore incorporates all conservative approximations and simplifications that might be implemented in actual receivers.

## 3. RAIM PERFORMANCE

In [DO-208], RAIM performance is measured in terms of *maximum allowable alarm rate and minimum detection probability (integrity)*, that both depend on

- Satellite failure rate
- Range accuracy
- Measurement geometry

Within the context of this paper, it will be assumed that the GPS range accuracy and the failure rate of the satellites are known. In that case, RAIM performance depends on geometry only, and RAIM availability prediction really becomes geometry screening.

The performance parameters are usually specified in terms of probabilities per flight hour. For analysis of a navigation system, it is useful to translate these to the probabilities that certain events (such as 'there is a position failure' or 'an error is detected') occur in a certain sample[i]. These probabilities and their dependence on geometry are discussed in the coming paragraphs.

### 3.1 Maximum alarm rate

Because TSO-C129 receivers are not required to do failure exclusion, every detection of an error will cause an alarm. Detection can occur due to noise, while all satellites operate within specifications, or can be the result of satellites failure(s):

$$P_{alarm} = P_{noise\_induced\_alarm} + P_{failure\_induced\_alarm}$$

The probability that noise causes an alarm while all satellites function properly can be written as

$$P_{noise\_induced\_alarm} = P_{all\_sats\_OK} \cdot P_{alarm|all\_sats\_OK} \quad (3.1)$$

when $P_{alarm|all\_sats\_OK}$ is the probability that an alarm is raised given the absence of satellite errors. An alarm can also be the result of a failure of one out of the $N$ satellites in view, with an associated probability of:

$$P_{failure\_induced\_alarm} = P_{sat\_failure} \sum_{i=1}^{N} P_{alarm|sat\_failure,i} \quad (3.2)$$

in which $P_{sat\_failure}$ is the probability that a satellite is in failure, and $P_{alarm|sat\_failure,i}$ is the probability that an alarm is raised given the failure of satellite $i$.

Because the probability that none of the satellites in view is failing simply equals:

$$P_{all\_sats\_OK} = (1 - P_{sat\_failure})^N \quad (3.3)$$

the probability of alarm can be written as:

$$P_{alarm} = (1-P_{sat\_failure})^N \cdot P_{alarm|all\_sats\_OK} + P_{sat\_failure} \cdot \sum_{i=1}^{N} P_{alarm|sat\_failure,i} \quad (3.4)$$

Expression (3.4) can be considerably simplified. Because TSO-C129 receivers are tuned to detect at least 99.9% of all significant satellite failures, almost all satellite failures will be detected and we can set $P_{alarm|sat\_failure,i} \approx 1$. Furthermore, the probability of a satellite failure is very low, and thus $(1-P_{sat\_failure})^N \approx 1$. Both these approximations are quite accurate and give slightly conservative alarm rates. Applying them gives the following expression for the probability of alarm:

$$P_{alarm} = P_{alarm|all\_sats\_OK} + N \cdot P_{sat\_failure} \quad (3.5)$$

TSO-C129 receivers are allowed to an alarm rate of $6.67 \cdot 10^{-5}$ per sample while the probability of the tracking a failing satellite failure is about $4.8 \cdot N \cdot 10^{-7}$ per sample[ii]. Therefore, the influence of satellite failures is relatively small and is often neglected to simplify (3.5) to

$$P_{alarm} = P_{alarm|all\_sats\_OK} \quad (3.6)$$

It is this equation (3.6) that is usually proposed to set the error detection threshold [Brenner90, Sturza88, Brown92, Leva96].

### 3.1.1 Alarm rate and AUGUR

From the previous considerations, the question arises which alarm probability expression should be used in AUGUR. Although most literature references use (3.6), this gives slightly optimistic alarm rates and RAIM availability. AUGUR therefore assumes that receivers use equation (3.5) instead of (3.6). This will make the prediction slightly conservative when receivers actually use (3.4) or (3.6), which is in line with the JAA requirements.

## 3.2 System integrity

The integrity of a system is affected any time the error in the position exceeds the maximum allowed error, the so-called horizontal alarm limit (HAL), while the RAIM error detection algorithm raises no alarm. In that situation, the system is said to provide Hazardous Misleading Information (HMI). In general, this can happen both in the absence and presence of a satellite failure. However, for en-route application, the probability of a position error caused by noise only is negligible as the nominal system accuracy is many orders of magnitude better than the maximum allowed position error. When the system meets the required 95% accuracy performance (100 meter) of [DO-208], the probability of HMI becomes virtually zero. Therefore only the situations with satellite failures will have to be taken into account, and the expression for the probability of HMI can be written as:

$$P_{HMI} = P_{sat\_failure} \sum_{i=1}^{N} P_{HMI|sat\_failure,i} \quad (3.7)$$

in which the probability of HMI given the failure of the $i^{th}$ satellite can be decomposed into:

$$P_{HMI|sat\_failure,i} = P_{pos\_error|sat\_failure,i} \cdot P_{no\_alarm|sat\_failure,i} \quad (3.8)$$

in which:

$P_{pos\_error|sat\_failure,i}$ : Probability of a position error in case of a failure in satellite *i*.

$P_{no\_alarm|sat\_failure,i}$ : Probability of no alarm in case of a failure in satellite *i*.

## 3.3 The system model

The previous paragraphs have shown that the system's alarm (3.5) and missed detection probability (3.7) can be computed from the probabilities of the following three events:

1. An alarm is raised in the absence of satellite failures
2. An alarm is raised in case of a satellite failure
3. A position error occurs in case of a satellite failure

These probabilities can only be computed within the context of a specified GPS system model. Although the real system model is non-linear, virtually all literature on RAIM algorithms seems to assume that the relation between the GPS measurements and the user position is sufficiently well described by a linearised overdetermined regression model:

$$\vec{z} = H \cdot \vec{x} + \vec{v} \quad (3.9)$$

in which:

$\vec{z}$ : *N*-vector of measurements

$H$ : $N \times M$ observation matrix with $N>M$

$\vec{x}$ : *M*-vector of unknowns (position, clock bias)

$\vec{v}$ : *N*-vector with independent noise and biases in the measurements, normally distributed as $\vec{v} \sim N(\vec{\mu}_v, \sigma^2_{GPS} I)$

It is important to stress that the model (3.9) still contains an unspecified parameter: the mean noise vector $\vec{\mu}_v$. Unfortunately, there is no full knowledge of $\vec{\mu}_v$ present. When all satellites operate well, it is about zero. When on the other hand the $i^{th}$ satellite is in failure, the corresponding element in $\vec{\mu}_v$ gets some unknown nonzero bias $B_{sat,i}$:

$$no\ \ failures \quad \Rightarrow \vec{\mu}_V = \vec{0}$$
$$failure\ in\ sat.i \Rightarrow \vec{\mu}_V = [0 \ \ldots \ 0 \ \underset{i}{B_{sat,i}} \ 0 \ \ldots \ 0]^T \tag{3.10}$$

There are two strategies to deal with the unknown bias error $B_{sat,i}$, that will both be discussed later.

The receiver uses the model (3.9) to compute both a position estimate and an error detection test statistic. The following two paragraphs will discuss the computations involved, and also show how the related probabilities of a position error and of error detection can be determined.

### 3.4 Position error probability

Due to the overdetermination, different approaches could be followed to compute a position estimate. Fortunately, there seems to be agreement that the best way to proceed is to use as many measurements as possible in the position estimate[iii] and use a simple least squares position estimation scheme. The resulting equation for the estimated position becomes:

$$\hat{\vec{x}}_{LS} = H^+ \vec{z} \tag{3.11}$$

in which $H^+$ is the Moore-Penrose pseudo-inverse of $H$ [Golub96], defined as:

$$H^+ = (H^T H)^{-1} H^T \tag{3.12}$$

The position error $\Delta\vec{x}$ that results is normally distributed according to:

$$\Delta\vec{x} = \vec{x} - \hat{\vec{x}}_{LS} \sim \mathrm{N}\left( H^+ \vec{\mu}_V, \sigma_{GPS}^2 \left(H^T H\right)^{-1} \right) \tag{3.13}$$

Figure 1 shows how the position error probability is to be computed. Because its underlying distribution is normal, points of equal probability density lie on ellipsoids centred on the mean position error (bias) $H^+ \vec{\mu}_V$. The probability of a position error is the content of the area of the error ellipsoid that falls outside the circle with a radius equal to the horizontal alarm limit. The exact probability can be found by integrating the probability density function (3.13) over the shaded area. Although straightforward, computation of the probability content of this area is quite involved [Ober97]. It is therefore often approximated by simpler distributions, as will be described in section 4.2.

Before discussing the probability that an error is detected, it will be useful for future reference to introduce the following observation on the relation between satellite bias and user position bias. From (3.10),(3.11) and (3.13) it is readily seen that for an error in satellite $i$, the size of the position error *when the noise is neglected* can be written in the form

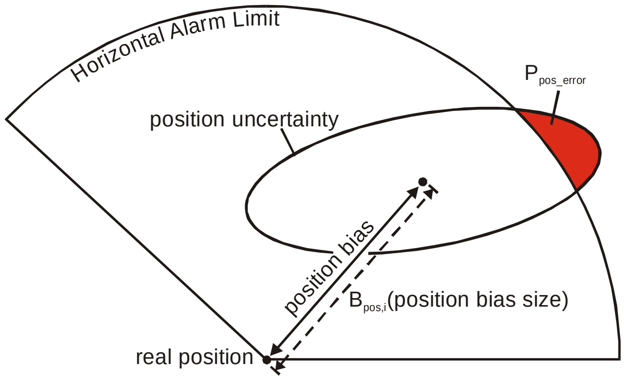


Figure 1. *The position error probability is the probability content of the position error distribution outside the allowed circle.*

$$B_{pos,i} = \left\| \Delta\vec{x} \right\| \Big|_{no\ noise} = c_{sat2pos,i} B_{sat,i} \tag{3.14}$$

where $c_{sat2pos,i}$ is some positive constant that depends on the elements in $H^+$.

### 3.5 The probability of a detection

All RAIM error detection tests published use the least squares residual[iv] that is nothing but an estimate of the measurement errors and noise. The residual can be written as

$$\hat{\vec{v}}_{LS} = \left(I - HH^+\right)\vec{z} \tag{3.15}$$

and is normally distributed as:

$$\hat{\vec{v}}_{LS} \sim \mathrm{N}\left(\left(I - HH^+\right)\vec{\mu}_V, \sigma_{GPS}^2 I\right) \tag{3.16}$$

The position error (3.13) and the least squares residual (3.15) are statistically independent (for a proof, see [Ober97] ). As a result, so are $P_{pos\_error|sat\_failure,i}$ and $P_{no\_alarm|\ sat\_failure,i}$ from (3.8) .

Based on the least squares residual, there have been described two different test statistics[v] for error detection:

- the (squared) $L_2$ norm of the residual $\left\|\vec{v}_{LS}\right\|_2^2$
- the maximum residual ($L_\infty$ norm) $\left\|\vec{v}_{LS}\right\|_\infty$

Note that the maximum residual is the defined as the largest of the *absolute values* of all elements in the residual vector, which is nothing but the vector's $L_\infty$ norm. Because the residual's squared norm has a chi-squared distribution, its use will be referred to as applying the 'chi-squared algorithm'. This algorithm is used in many references such as [DO208,Brown92,Sturza88, Leva96].

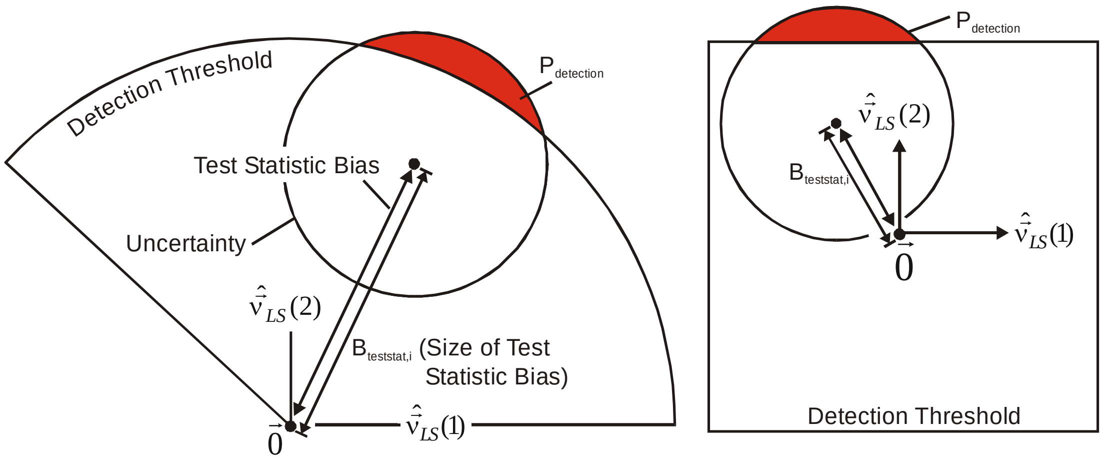


Figure 2. *The two different decision rules for error detection. On the left hand side, an error is detected when the least squares residual lies outside a circle (chi-squared algorithm), on the right hand side when it lies outside a box (maximum residual algorithm). In both figures, the N-dimensional residual space has been depicted in two dimensions only.*

Just as in the case of the position error, for the noiseless case the relation between the satellite bias and the norm of the least squares residual can be written in the form:

$$B_{teststat,i} = \left\| \hat{\vec{v}}_{LS} \right\|_{2\ or\ \infty} \Big|_{no\ noise} = c_{sat2teststat,i} \cdot B_{sat,i} \qquad (3.17)$$

in which $c_{sat2teststat,i}$ is the $i^{th}$ diagonal element of the matrix $I\text{-}HH^{+}$.

In both algorithms, an alarm is raised as soon as the least squares residual lies outside a specified area. This area is a circle for the chi-squared algorithm and a box for the maximum residual algorithm, see Figure 2. The probability of detection can be determined by integrating the probability density function of the residual over the detection area. The probability content of this area is described by standard distributions: either a (central or non-central) chi-squared distribution function for the chi-squared algorithm or a Gaussian (normal) distribution for the maximum residual algorithm[vi]. Therefore, standard tables can be used.

### 3.5.1 Detection probability and AUGUR

[Kelly97] shows that the maximum residual algorithm has a slightly superior error detection performance for the same alarm rate. The extra power is obtained because the algorithm exploits the knowledge on both size and direction of the least squares residual, while the chi-squared algorithm uses only the size. In view of the JAA requirements, AUGUR remains conservative and therefore assumes that the receiver uses a chi-squared algorithm for error detection.

## 3.6 Dealing with the unknown bias

In the previous discussion, the computation of RAIM performance parameters has been described. The methodology described is 'exact' and does not contain any approximations or simplifications yet. One problem has not been covered yet: in case of a satellite error both the probability of a position error and an alarm depend on the unknown bias $B_{sat,i}$ in the failing satellite. To stress this dependence, (3.7) can be written explicitly as a function of $B_{sat,i}$:

$$P_{HMI}(B_{sat,i}) = P_{pos\_error}(B_{sat,i}) \cdot P_{no\_alarm}(B_{sat,i}) \qquad (3.18)$$

Note that for notational convenience, the fact that these probabilities are conditioned on the presence of a satellite failure has been dropped.

When the bias in a failing satellite grows, so do the biases in the position and test statistic, increasing the probabilities of a position error but also of detection. Because the parameter $B_{sat,i}$ is unknown, some value will have to be substituted to enable performance computation. Two strategies are known:

1. Substitute minimal detectable biases
2. Substitute worst case biases

These methods will be discussed in the next two paragraphs.

### 3.6.1 Minimal Detectable Biases

The minimal detectable bias (MDB) principle (e.g. [Sturza90,Leva96]) approaches the 'unknown bias' problem as follows. The TSO-C129 requirements state, that position errors have to be detected with a probability of at least 99.9%. For this requirement, the minimal detectable bias is defined as *the smallest satellite bias that can be detected with at least 99.9% probability*. Satellite biases smaller than the MDB will be detected with a lower probability.

Instead of computing the missed detection probability, the MDB approach usually continues by computing the '*maximum undetectable position bias*' that is detected with less than 99.9% probability (by substituting the MDB for $B_{sat,i}$ in (3.18)). RAIM is then declared available when this bias is smaller than the maximum allowed position error. Note that in this process, except for the centre, the position error distribution remains unused! This is due to the fact that it is implicitly assumed that all undetected satellite failures will cause a position failure and $P_{no\_alarm}(MDB)$ is used as an upperbound on the probability of *HMI*:

$$P_{HMI}(MDB) = P_{pos\_error}(MDB) \cdot P_{no\_alarm}(MDB) < P_{no\_alarm}(MDB) \qquad (3.19)$$

At first sight, the MDB approach therefore seems to be conservative, as not all undetected satellite failures will cause position failures. A significant part of the position error distribution will lie in the area where the position error is within the required bounds. Therefore, the probability that a position failure is undetected will always be significantly higher than 99.9%.

The previous reasoning is perfectly true, *as long as the satellite bias indeed equals or exceeds the MDB.* However, satellite biases smaller than the MDB can still cause HMI probabilities exceeding 0.1% due to their reduced detectability (higher $P_{no\_alarm}$). For certain combinations of satellite geometry and satellite errors, the use of the MDB *can* be an underestimation of the probability of HMI!

The situation is depicted symbolically in Figure 3. The solid line in this figure represents the relation between the size of the position bias and the size of the test statistic bias:

$$B_{pos,i} = \frac{c_{sat2pos,i}}{c_{sat2teststat,i}} \cdot B_{teststat,i} \qquad (3.20)$$

The 'cloud' that is drawn as an ellipsoid (although not a real ellipsoid in practice) represents the uncertainty that is introduced by the presence of noise. It's centre lies in ($B_{pos,i}$, $B_{teststat,i}$) and shifts towards the upper-right for a growing satellite bias. The MDB is the value of the satellite bias that causes exactly 99.9% of the noise cloud to lie beyond the error detection threshold $T_{threshold}$.

### 3.6.2 Worst Case Satellite Biases

The possibility that the MDB approach could be underestimating the missed detection probability for certain satellite biases has been realised for some years now [Brown94,Lee95,Ober97]. Instead of only considering the detectability of a bias, as done in the MDB approach, these references all take both the detectability and the chances of causing a position error into account. Although the way to accomplish their goal differs, they all explicitly solve for a *worst case bias* that maximises the missed detection probability, as illustrated in Figure 3b. Unfortunately, the computation of the worst case bias for a particular geometry is computationally involved. The Approximate Radial error Protected (ARP) method [Chin92] has used Monte Carlo simulation with many different geometries to determine the worst case probability of HMI (and therefore implicitly the worst case satellite bias). It is unclear whether the values found in [Chin92] are representative for every geometry and every satellite constellation, or are only valid within the context of the optimal 21 satellite constellation used in the simulations.

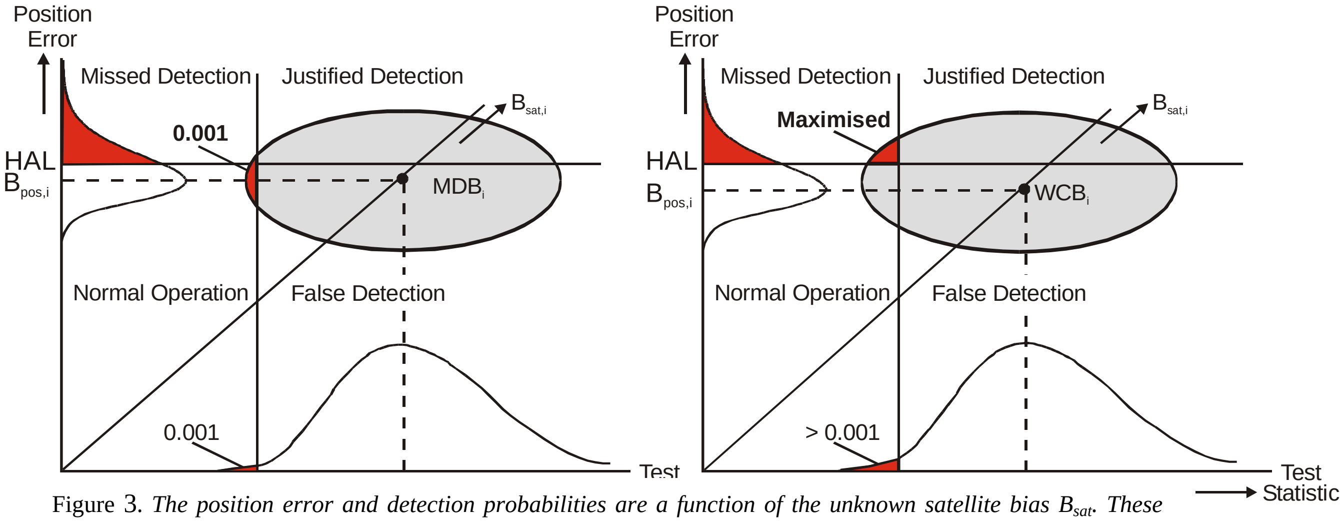


Figure 3. *The position error and detection probabilities are a function of the unknown satellite bias $B_{sat}$. These figures show the situations when the unknown bias is replaced by the Minimum Detectable Bias (left hand side) or Worst Case Bias (WCB, right hand side).*

### 3.6.3 Unknown biases and AUGUR

In the implementation of the AUGUR algorithm, it would have been appropriate to take the worst case bias approach because the MDB *might* lead to overly optimistic results. Unfortunately, the computational complexity of finding the worst case bias iteratively prevented the use of this approach, but this is most likely also true for current TSO-C129 receivers. Using the ARP method is a rather unsatisfactory alternative – although it uses worst case biases, it has the disadvantage of using Monte Carlo derived parameter settings that are only valid 'in the mean' and for the optimal 21 satellite constellation.

When comparing ARP to MDB, it turned out that for the given requirements and satellite noise, use of MDB leads to an earlier rejection of a geometry than ARP does. This means that for the case at hand, MDB is more conservative than ARP is. AUGUR therefore uses the MDB approach.

## 4. SIMPLIFICATIONS

The previous section has discussed the computations of the integrity and alarm rate as a function of geometry. The problem of exact computation is well posed, but computationally involved, especially when the worst case bias approach is used. Moreover, there are many different parameters involved. It is often convenient to reduce the number of parameters involved to arrive at simpler decision criteria for RAIM availability and reduce the computational effort. This section will describe which simplifications are often used and what the consequences are in terms of performance.

### 4.1 Least detectable satellite only

Often, the assumption is made that a satellite failure, when it occurs, occurs on the satellite that has the smallest error detection probability for a bias of a certain size. In fact, many researchers interpret the following phrase out of [DO208] as a *demand* to consider only this worst satellite:

> "*The integrity system shall meet the specified detection probability globally at all times for single satellite failures, except for those conditions where integrity cannot be assured and the flag is displayed accordingly*".

Likewise, the TSO-C129 RAIM tests are performed by putting a ramp bias on the least detectable satellite and therefore seem to be based on this assumption. In reality, every satellite is equally likely to fail, and it should therefore be clear that considering this 'worst case' satellite only leads to an overestimation of $P_{HMI}$ and an underestimation of RAIM availability. However, assuming that it is only the least detectable satellite that fails, the expression for $P_{HMI}$ (3.7) can be simplified to read:

$$P_{HMI} = N \cdot P_{sat\_failure} \cdot P_{pos\_error|sat\_failure,LDS} \cdot \cdot (1 - P_{detection|sat\_failure,LDS}) \tag{4.1}$$

where *LDS* is the index of the least detectable satellite. Which of the satellites is least detectable has yet to be determined. In has already been shown that in the noiseless case there are simple linear relations between the satellite bias and resulting position and test statistic (3.14) and (3.17). As a result, the test statistic and position error are also linearly related by:

$$B_{pos,i} = SLOPE_i \cdot B_{teststat,i} \tag{4.2}$$

in which the $SLOPE_i$ parameter is introduced as:

$$SLOPE_i = \frac{c_{sat2pos,i}}{c_{sat2teststat,i}} \tag{4.3}$$

Referring once more to Figure 3, it should be clear that a higher value of $SLOPE_i$ corresponds to a larger probability of HMI, because a larger part of the noise cloud will lie in the missed detection area. Therefore, the satellite with the highest value of $SLOPE_i$ is usually considered least detectable.

Note that there is a potential problem in using only the $SLOPE_i$ value that is obtained from a noise-free approximation. It might be that there are cases when the influence of noise makes another satellite least detectable. The exact position error distribution and therefore the shape of the noise cloud in Figure 3 changes differently for growing failures on different satellites. This *could* make the worst case bias on a satellite with a lower $SLOPE_i$ value result in a higher probability of HMI than the worst case bias on the satellite with the highest $SLOPE_i$[vii]. To the best of the author's knowledge, it is yet unknown to what extent the actual RAIM performance can vary for equal values of $SLOPE_{MAX,}$ and how reliable the use of the $SLOPE_{MAX}$ really is.

#### 4.1.1 Representing geometry by HDOP

In the example algorithm of Appendix F of [DO208], the geometry is represented by the largest *HDOP* of a subset of 4 out of 5 satellites (called $HDOP_{max}$ here). This is another way of representing a geometry by one 'worst case satellite' related figure.

It has been shown that this is an extremely poor way to do geometry screening [Chin92]. Chin states:

> "*… we conclude that $HDOP_{max}$ is not a useful criterion*".

Because of the poor correlation of its value with the RAIM performance parameters, use of $HDOP_{max}$ is shown to reduce the availability of RAIM largely due to the resulting need for high margins and the difficulty of determining a suitable threshold for unavailability.

#### 4.1.2 Least detectable satellites and AUGUR

The assumption that it is always the least detectable satellite that fails is obviously very conservative. Still, it is widely used and applied [Brown90, Leva96, Sturza90, Lee95, Kelly97], and AUGUR incorporates it as well. All references use the $SLOPE_i$ rather than the unreliable $HDOP_{max}$ criterion. Although it is certainly possible that there are receivers that still use the latter criterion, RAIM availability would be unnecessarily degraded when AUGUR would do this as well. Furthermore, there are no solid criteria to choose a $HDOP_{max}$ threshold. For this reason, AUGUR uses the $SLOPE_i$ criterion instead.

### 4.2 Approximate probability densities

The exact computation of the probability of a position error involves integration of a two-dimensional normal probability density function over a circle. The exact computation of detection probabilities needs the probability function of a non-central chi-squared distribution. Both exact computations are computationally involved, and throughout RAIM literature different approximations have been used. Three of them will be discussed shortly. Details and proves can be found in [Ober97].

#### 4.2.1 Normal approximations

The noncentral chi-squared distribution of the $L_2$-norm of the least squares residual (or parity vector) can be approximated by a normal distribution, that is in fact the marginal distribution of the residual along a certain line. This approach is for example used in [Lee95], and leads to an underestimation of the probability of detection. Therefore, this approximation is conservative as it leads to earlier rejection of geometries. In a similar way the position error can be approximated normally, as has been advocated in [Lee95] and [Kelly97]. It has the disadvantage of *underestimating* the probability of a position failure in some cases, and can lead to geometries being accepted that should be inadmissible.

#### 4.2.2 Chi-squared approximation of the probability of a position failure

To avoid the underestimation of the position error, the distribution of the position error can be bounded by a non-central chi-squared function [Ober97]. This method always *overestimates* the probability of position error, leading to a conservative RAIM availability.

#### 4.2.3 Approximate densities and AUGUR

AUGUR does not use the discussed approximations of the position error probability, as it employs the MDB approach. This makes the exact probability density of the position error immaterial, as the MDB approach is based on the assumption that the position error probability simply equals one as soon as an undetected satellite failure is present (3.19). Normal approximations to the probability of *detection* could still be considered, as this would lead to results that are more conservative. However, it has been considered very unlikely that manufacturers would use this approximation. First of all, the MDB method allows computation of the threshold-setting offline, making the gain in computational speed of the approximation immaterial. Moreover, all literature references that discuss the MDB method use exact chi-squared distributions for the test statistic as well. Therefore, so does AUGUR.

## 5. HOW ALGORITHMS DIFFER

Using the material in the previous sections, the question concerning the possible different ways to implement RAIM error detection and RAIM availability calculations can be addressed. In the following overview, first all previously covered items will be summarised. Then, some other, more obvious but still important potential differences are discussed, such as parameter settings, operational conditions and receiver limitations.

### 5.1 Algorithmic differences

RAIM detection and geometry screening algorithms can differ in a couple of ways that have been discussed in detail before. Either the “chi-squared” or the “maximum residual” algorithm can be used. Furthermore, two different methods for substitution of the unknown bias in case of a satellite failure can be exploited, based on either a minimal detectable or a worst case bias principle. Especially when the worst case bias is used, many different ways exist to simplify the complicated probability distributions of position and test statistic.

All known algorithms represent the geometry by the least detectable satellite only, even though this results in conservative availability figures. Although the foundation for determining the least detectable satellite is not entirely sound, the $SLOPE_i$ criterion seems to be the only reliable criterion that is currently available.

### 5.2 Parameters settings

Naturally, algorithms can differ in the values chosen for the parameters that have been assumed fixed in the discussion so far:

- GPS/barometer range accuracy
- failure rate of GPS satellite/barometer

Obviously, higher values of these parameters would lead to a decrease in RAIM availability. The way of calibrating the barometer (by GPS or local pressure) and the use of filtering of the GPS pseudoranges to reduce noise influence the performance of the algorithm as well.

### 5.3 Geometric differences

Although the GPS constellation is the same for all receivers that are operated at the same time at the same geographic position, they might still experience different measurement geometries because they track different satellites. Important factors to be considered are:

- mask angle
- number of satellites tracked (or used) for RAIM and positioning
- criteria used for subset selection: [DO208] proposes to select 5 satellites based on best accuracy considerations, while other sources [Graas93] propose a best integrity criterion instead
- the use of barometric aiding

Note that the number of satellites that are tracked during operation might also change for different flight levels and bank angles.

## 6. THE AUGUR ALGORITHM SUMMARISED

In summary, the JAA requirement for the AUGUR RAIM prediction software to be either exact or conservative has been translated in the use of:

- a chi-squared distributed error detection test statistic
- representation of the satellite geometry by the least detectable satellite only
- use of the minimal detectable bias principle to cope with the unknown behaviour of erroneous satellites

As has been shown, all choices in the algorithm design are conservative without degrading the RAIM availability to an unreasonable extent.

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[i] In the translation from 'event per hour' to 'event per sample', it is usually assumed that an independent sample can be taken once every 2 minutes due to the correlation time of selective availability, see [DO208].

[ii] Value based on the failure rates of [SPS]. [DO208] uses a lower failure rate of about $2.1 \cdot N \cdot 10^{-7}$ / sample

[iii] In principle, one could decide to use as few as 4 measurements in the position determination only. This would lead to a decreased accuracy, but has the advantage that errors in the satellites that are not used do not influence the position

[iv] The least squares residual is an *N*-vector that is restricted to lie in an *N*-4 dimensional space. Therefore, often a so-called ‘parity vector’ is used (obtained by a simple co-ordinate transformation), that is an (*N*-4)-vector with the same information content as the least squares residual but with independent elements. Whether the least squares residual or the parity vector is used is only a matter of convenience, as their statistic properties are identical, see also [Brown92].

[v] Most other techniques only differ from the two above ‘on the surface’, while being functionally equivalent. [Kelly97] shows that the well-known algorithm by Brenner [Brenner90] equals the maximum residual algorithm, while the ‘single deletion’ algorithm of [Parkinson88] matches the chi-squared algorithm.

[vi] The Gaussian distribution is not exact but rather gives an underbound on the detection probability, see for example [Barnett94] for details.

[vii] It might be possible to prove or disprove this statement. Author is not aware of the any such prove. As safety is involved, doubt should remain until a solid proof is found.