# Three High Integrity Positioning Algorithms

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**ABSTRACT**

The paper investigates the integrity performance of three different position estimation algorithms that have been designed following the High Integrity Positioning (HIP) concept. Their performances are compared to both each other and the traditional least squares algorithm. Thus it is shown that the HIP concept offers definite advantages in terms of the integrity that can be achieved.

## 1. INTRODUCTION

Integrity and continuity requirements are important drivers of GNSS user equipment system design. Especially in safety critical operational environments, it is importance to exploit all available information to assess position integrity as good as possible - to avoid both being optimistic (compromising integrity) and pessimistic (sacrificing continuity).

While position integrity is often the crucial parameter of interest, traditional positioning algorithms are generally optimised for accuracy instead and have to be combined with fault detection and exclusion schemes. Still, these algorithms give sub-optimal integrity. A new approach to algorithm design has therefore been introduced in [Ober, 1999] that takes integrity rather than accuracy as the parameter to optimise. This technique is referred to as High Integrity Position (or Parameter) estimation (HIP).

The paper will first review the main ideas behind HIP. A recently developed Bayesian position domain methodology for integrity computation will then be exploited to compare the integrity of traditional least squares schemes with three different HIP estimators. A small-scale simulation shows that the HIP concept can indeed provide optimised integrity and availability.

## 2. HIGH INTEGRITY POSITIONING

Current positioning algorithms are almost exclusively based on least squares estimation schemes [Leva, 1996][Brown, 1996]. When the system is linear and measurement noise is normally distributed, this gives optimal accuracy: least squares is the best way to mitigate the effects of noise on the position. The obtained position is however sensitive to failures: a single wrong measurement (modelled as a bias) can cause an arbitrarily large position error. To warn the user that a failure is present, error detection has to be used.

Error detection typically uses a test statistic based on the least squares residual, that in the absence of biases is the most accurate estimate of the measurement error. When the test statistic exceeds a certain threshold, an error is detected. It can be proven that the least squares residual and the position error are statistically independent [Ober, 1997]. The reason that detection still works is that both residual and position are influenced by the same deterministic bias. However, the noise in the position error is not reflected at all in the residual, indicating that the error detection properties – and therefore integrity- might not be optimal. In order to optimise integrity, a new approach to algorithm design is required, that focuses on the relation between position error and error detection signal, rather than on positioning and error detection separately. This concept has been introduced in [Ober, 1999] and is called high integrity positioning (HIP).

In this paper, the integrity performance of three different HIP based algorithms is going to be compared. The algorithms will be evaluated by a relatively new Bayesian approach to assess integrity performance [Ober, 2000] that will simultaneously be the basis for two of the three HIP estimators *and* for the generation of integrity alarms.

## 3. BAYESIAN ASSESSMENT OF SYSTEM INTEGRITY

In [Ober, 2000] it is shown in detail how the statistical distribution of the position can be derived from the measurement data as a weighed sum of error distributions, in which the weights depend both on the measurement residuals, the statistical properties of the measurements and the a priori probabilities of failure. In this section, the main results will be summarised.

The system is assumed to be sufficiently well described by an overdetermined ($N > M$) set of linear equations that relate the measurements to the unknown parameters as

$$\underline{z} = H\,\underline{x} + \underline{n} + \underline{b} \tag{3.1}$$

with

$\underline{z}$ : $N$-vector of measurements
$H$ : $N \times M$ observation matrix
$\underline{x}$ : $M$-vector with unknown parameters
$\underline{n}$ : $N$-vector of measurement noise
$\underline{b}$ : $N$-vector of measurement biases

As only single failures will be considered, it will prove convenient to use $E_i$ to denote the event of a failure in measurement $i$ and represent the no failure case by $E_0$. The probability of occurrence of each event is denoted by $P(E_i)$. Under the no failure case $E_0$, the measurements are assumed unbiased, while under $E_i$ the failing measurement contains an unknown bias.

When the noise is normally distributed, the information on the position that can be extracted from the measurements can be summarised in a random variable that is distributed as a mixture of $N+1$ normal distributions:

$$\tilde{\underline{x}} \sim \sum_{i=0}^{N} w_i N(\underline{\mu}_{x,i}, \Sigma_{x,i}) \qquad (3.2)$$

in which $\underline{\mu}_{x,i}$ and $\Sigma_{x,i}$ represent the relevant mean and covariance under each of the events $E_i$. The weights are very informative, and are a function of all important parameters: geometry, satellite failure rate and least squares residuals under each event. They become larger when:

1. the a priori probability $P(E_i)$ is higher
2. the residuals under $E_i$ are smaller

The *exact* probability of hazardously misleading information *P(HMI) for an arbitrary position estimate* is simply derived from (3.2) as that part of the distribution that lies outside the Horizontal Alarm Limit (see Figure 1). When *P(HMI)* exceeds the value specified in the integrity requirements, an alarm can be raised to the user. Note that this means that the alarm criterion is defined in the position domain, while traditionally alarm criteria are based on range-domain parameters such as the least squares residual (or parity vector). Another observation is that the expression (3.2) downweights measurements with high a posteriori probabilities of being biased. It therefore implicitly performs fault exclusion.

*P(HMI)* expresses the integrity risk and measures the integrity of the position solution. It will therefore be used to assess the integrity of the different estimators that will be discussed.

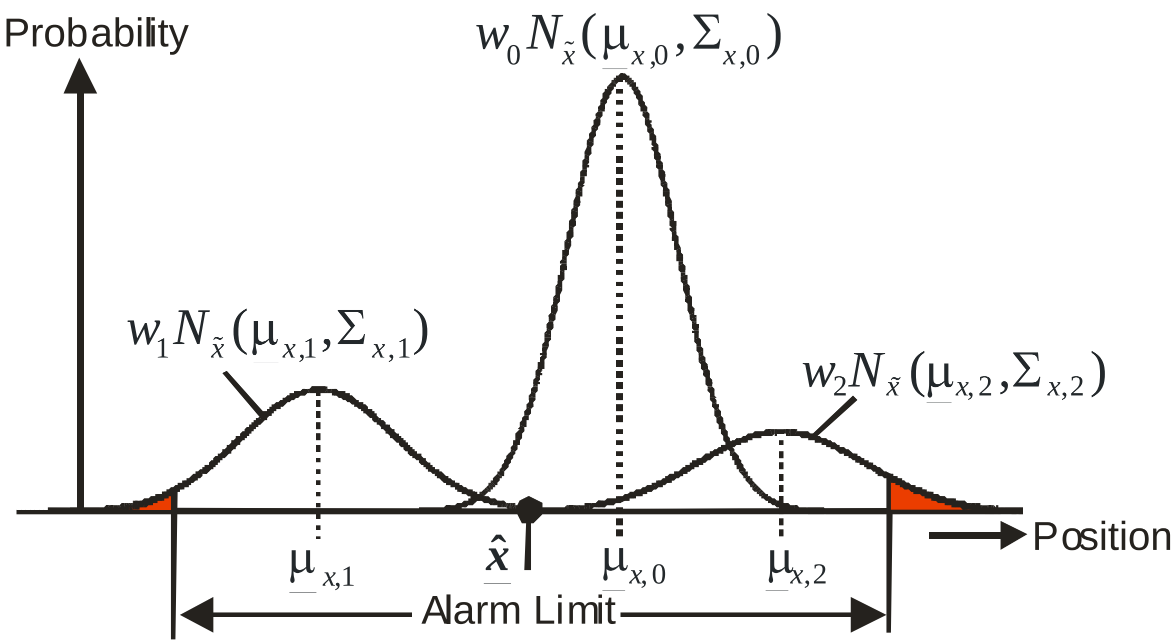


Figure 1. The distribution of the position as a function of the measurements and the geometry

## 4. THREE HIP ESTIMATORS

Based on the HIP concept, three different estimators have been designed, of which this section gives a short description.

1. The first HIP method is based on the approximate (Horizontal) Protection Level from [Leva, 1996]. When the position error that is introduced by the 'minimal detectable bias' due to a failure of satellite *i* is written as $PL_i$, the Protection Level is taken as the error introduced by the worst case satellite as:

   $$PL = \max_i PL_i$$

   The estimator that minimises *PL* is the first HIP estimator, to be indicated as $HIP_1$; some results of its performance can be found in [Ober, 1999]. It can be computed iteratively and is guaranteed to converge to a global minimum.

2. The second HIP method is based on a full minimisation of the integrity risk *P(HMI)* and therefore also functions as a benchmark as it gives the *best possible integrity*. In practice, $HIP_2$ has the disadvantage of being iterative and more computationally expensive than $HIP_1$. Furthermore, there might be multiple local minima that complicate the optimisation process.

3. A computationally more attractive solution is to use the expected value of (3.2) as the parameter estimate of interest. This gives the $HIP_3$ estimator. Due to the behaviour of the weights, this implies that the event $E_i$ that is most likely to have been occurred given the measurements will dominate the position estimation, that therefore automatically more or less rejects measurements that are suspected to be biased. Intuitively, this implies that they should have a build in integrity that should not differ too much from the optimal value.

Note that as they are based on (3.2), both $HIP_2$ and $HIP_3$ estimator contain fault exclusion functionality due to the downweighting of suspect measurements.

## 5. SMALL SIMULATION

A small simulation has been performed to obtain an impression of the performance improvements that might be obtained. Because of time limitations, it has been decided to postpone a full Monte Carlo study and look only at three different satellite constellations with 5, 6 and 7 satellites in view respectively. In all cases, a noise vector is added to the measurements with a length such that it lies on the edge of the area that contains 95% of the noise vectors and random direction. On the worst case satellite, a bias of growing size between 0 and 2150 meters is added while the noise vector is kept constant to enable to compute integrity in terms of (3.2) as a function of the measurement bias. Arbitrarily, a horizontal cross-track error of 300 meters has been chosen. A GPS range sigma of 33.3 meters [RTCA DO-208] has been used, while the satellite failure rate from [GPS-SPS] has been applied. The results in Figure 2 have been averaged over 100 different noise vector cases.

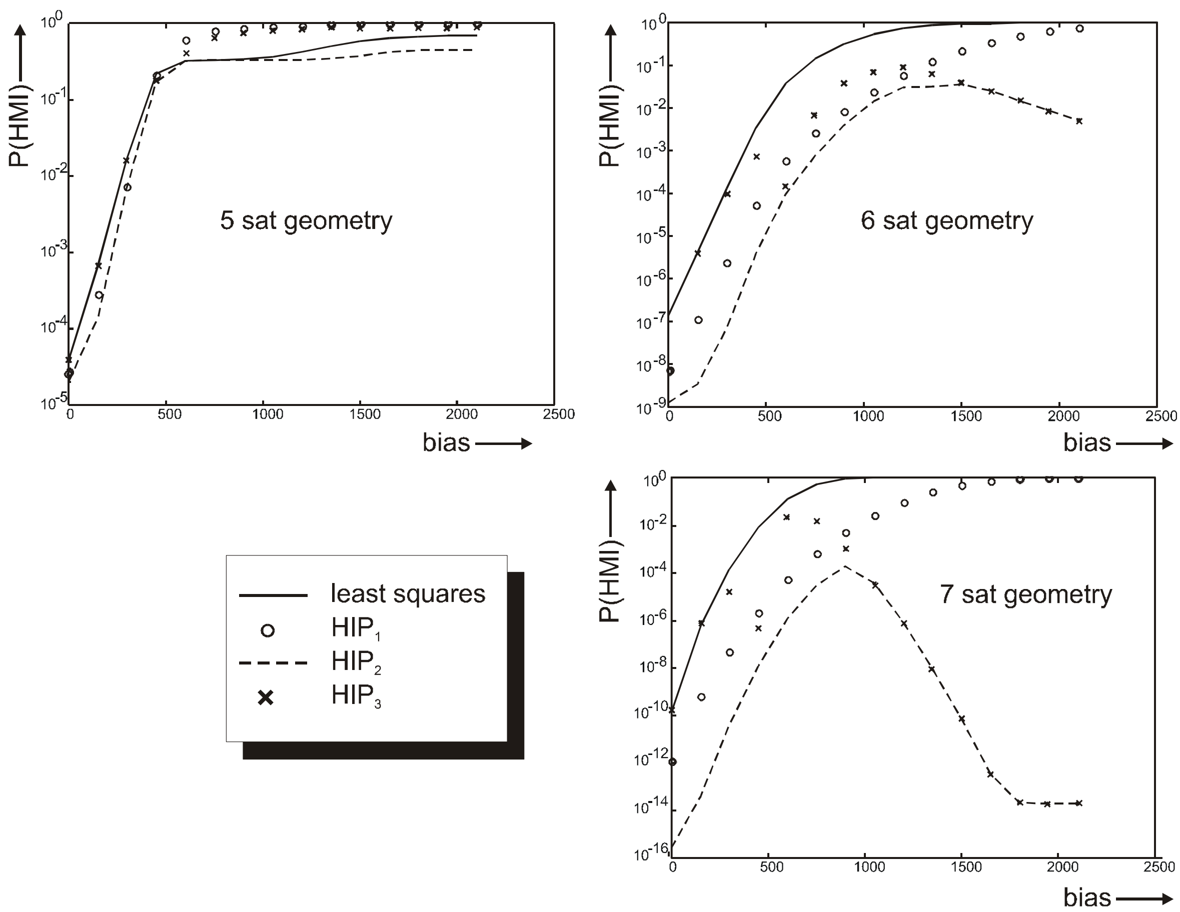


Figure 2. The probability of an error outside of the allowed alarm limit as a function of the bias in the worst case satellite for both least squares and the three HIP estimators

In the 5 satellite case, a rather low integrity is achieved with each of the estimators. Least squares does perform better than $HIP_1$ and $HIP_3$ and achieves a performance that is only slightly worse than the best possible performance that is (per definition) achieved by $HIP_2$. One can conclude that in this case, the geometry is the limiting factor and there are insufficient degrees of freedom to find better estimators.

The 6 and 7 satellite cases are quite similar. One can clearly see the exclusion effect that occurs when large biases are introduced in $HIP_2$ and $HIP_3$. Their performances also become very similar. For smaller biases, least squares performance degrades at a much larger rate than $HIP_1$. Both methods are unrobust in the sense that their performance degrades largely for large biases, and their availability will suffer from the fact that P(HMI) will grow rapidly beyond the allowed limits due to the lack of such an exclusion effect.

In conclusion, one can say that the three HIP methods, and especially $HIP_2$ and $HIP_3$ have the potential gain significant performance when they replace current fault detection (and exclusion) schemes.

## 6. CONCLUDING REMARKS

The paper has investigates the integrity performance of three different position estimation algorithms that have been designed following the High Integrity Positioning (HIP) concept. Their performances have the potential to reduce the probability of hazardously misleading information drastically, and can thus increase availability. This certainly justifies a more extensive investigation than the one presented in this paper.

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