ALGORITHM SELECTION FOR AUGUR - EUROCONTROL's PREDICTIVE RAIM TOOL

P.B. Ober, Delft University of Technology
D.A.G. Harriman, STASYS Ltd
J. Wilde, STASYS Ltd

ABSTRACT - *This paper summarises the process of the selection of an algorithm for EUROCONTROL's RAIM prediction tool AUGUR, focussing on design issues rather than mathematical details, which can be found in [Ober98]. The selection process is driven by the JAA requirement that the predicted availability be conservative when actual RAIM implementations are unknown. Algorithms from open literature are analysed to arrive at a conservative algorithm without sacrificing availability unnecessarily*.

1. INTRODUCTION

Pilots that want to fly Basic Area Navigation in ECAC airspace using stand-alone GPS, possibly with barometer aiding[1], will be required to check the availability of RAIM before taking off. To enable them to do so, the RAIM availability prediction package AUGUR has been developed for EUROCONTROL by STASYS Ltd. General information on AUGUR is available in a complementary paper [Harriman98] elsewhere in these proceedings, while this paper focuses on the selection of the algorithm that has been implemented. AUGUR can be found at the Internet at http://augur.ecacnav.com or http://augur.us.ecacnav.com.

RAIM consists of two algorithms: an error detection algorithm, that detects actual satellite failures, and a 'geometry screening' algorithm. Geometry screening consists of computing the error detection performance as a function of the geometry and deciding whether this performance meets the requirements. Because there is no dependence on the actual GPS signals, the sufficiency of the geometry is predictable and can be computed in advance for any user position and time. In this paper, such computations will be referred to as RAIM availability prediction - or RAIM prediction in short.

The prediction algorithm to be used by AUGUR has to obey the JAA requirement from [TGL-2]: "*The programme should use either a RAIM algorithm identical to that used in the airborne equipment, or an algorithm based on assumptions for RAIM prediction that give a more conservative result*".

It is important to understand that AUGUR has a purely operational goal: it aims at determining whether user receivers are going to experience any RAIM outages during flight. In other words: AUGUR tries to predict the behaviour of receivers rather than looking at the sufficiency of the GPS geometry as such. Because [TSO-C129] – to which GPS receivers have to comply receivers for Instrument Flight Rules (IFR) flight - does not prescribe any RAIM prediction algorithm, the implementation in the receiver is unknown. To obey the JAA requirements, AUGUR must err on the safe side and give a conservative prediction for all receivers. GPS users, however, would not like to see availability sacrificed unnecessarily. The approach taken is therefore to be conservative within reasonable limits, given by the numerous algorithms and techniques that have been published in open literature. This paper gives an overview of these algorithms and shows how a suitable algorithm for AUGUR can be derived by taking conservative choices at different steps in the algorithm design. The algorithm selection process starts by showing how the exact performance of error detection algorithms can be determined. The second step is to review which approximations and simplifications have been proposed to arrive at practically applicable computations. The algorithm in AUGUR will incorporate all those approximations and simplifications that are conservative whenever practical.

[1] In most of the paper, use of the barometer is only implicitly assumed

2. RAIM PERFORMANCE

RAIM performance is measured in terms of two system parameters: alarm rate and missed detection probability. Both parameters depend on the satellite failure rate, the GPS range accuracy and measurement geometry. When the range accuracy and the failure rate of the satellites are assumed to be known, RAIM performance depends on geometry only. RAIM is therefore available as long as the measurement geometry offers sufficiently low alarm rate and missed detection probability. Its performance can only be computed within the context of a specified GPS system model. Although the real system model is non-linear, virtually all literature on RAIM algorithms seems to assume that the relation between the GPS measurements and the user position is sufficiently well described by an linearised overdetermined regression model. AUGUR uses this assumption as well. The measurement noise is usually modelled as normally distributed with zero mean. In case of a satellite failure, the measurement of the corresponding satellite is modelled to contain an (unknown) bias as well.

2.1 Alarm rate

As TSO-C129 receivers are not required to do failure isolation and removal, an alarm is raised whenever the error detection algorithm detects an error. Detection can occur due to noise only, while all satellites operate within specifications, or can be the result of satellite failures. If at most one satellite fails simultaneously[2], the TSO-C129 receiver is required to detect this with at least 99.9% probability. Assuming conservatively that *all* failures are detected, the alarm probability equals

$$P(alarm) = P(erroneous\ satellite\ in\ view) + P(noise\ induced\ alarm)$$

The [SPS] predicts that three satellite failures might occur every year for the whole 24 satellites GPS constellation. Assuming a 120 second correlation time due to selective availability, this value can be translated to a probability per sample to get

$$P(erroneous\ satellite\ in\ view) = 4.8 \cdot N \cdot 10^{-7} \text{ per sample when there are } N \text{ satellites in view}$$

According to the [TSO-C129] requirements, the maximum alarm rate is $6.67 \cdot 10^{-5}$ per sample. This means that the threshold of the error detection algorithm should be chosen such that the probability of a noise induced alarm probability should obey:

$$P(noise\ induced\ alarm) < (6.67 - 0.048 \cdot N) \cdot 10^{-5} \text{ per sample} \qquad (*)$$

Often, the contribution of erroneous satellites is neglected [Brenner90, Sturza88, Brown92, Leva96] and the error detection algorithm is tuned according to $P(noise\ induced\ alarm) < 6.67 \cdot 10^{-5}$. This leads to a slight overestimation of the system alarm rate and therefore AUGUR uses (*) instead.

2.2 Missed detection probability

The integrity of a system is affected any time the error in the position exceeds the maximum allowed error while the RAIM error detection algorithm raises no alarm. In that situation, the system is said to provide Hazardous Misleading Information (HMI). In principle, this can happen both with or without the presence of a satellite failure. However, for en-route application, the probability of a position error caused by noise only is negligible, as the nominal system accuracy is many orders of magnitude better than the maximum allowed position error. When the system meets the required 95% accuracy performance (100 meter), the probability of noise induced HMI is virtually zero. Therefore only the situations with satellite failures will have to be taken into account, and the expression for the probability of HMI can be written as:

$$P(HMI) = P(erroneous\ satellite\ in\ view) \cdot P(position\ failure) \cdot (1 - P(failure\ induced\ alarm))$$

---

2 The probability of 2 failing satellites is 100 times smaller than the allowed undetected failure rate for 8 satellites in view

due to the statistical independence of *P(position failure)* and *P(failure induced alarm)* [Ober97].

When the position is computed from the measurements by least squares estimation, the normal distributions and linear model give a normally distributed position error distribution. This is elliptically shaped and is centred on the position bias, which depends linearly on the measurement bias of the failing satellite. *P(position failure)* is the content of the area of the error ellipsoid that falls outside the circle given by the allowed error range, see Figure 1a. The exact probability can be found by integrating the probability density function over the shaded area. Although straightforward, computation of the probability content of this area is quite expensive [Ober97]. It is therefore often approximated by simpler distributions.

For failure detection, all RAIM error detection tests published use the least squares residual[3] that is nothing but an estimate of the measurement errors and noise. The residual is normally distributed around a bias caused by the measurement bias in the failing satellite. The larger the residual becomes, the more likely that there is a bias (error) in the system. Error detection algorithms therefore define an "error detected" region and declare an error occurrence as soon as the residual lies in that region. Two choices for this region have been proposed: either a circle [DO208,Brown92,Sturza88,Leva96], see Figure 1b, or a square around the origin [Kelly97]. The former leads to an alarm probability that is described by the noncentral chi-squared distribution, and its use will therefore be referred to as applying the 'chi-squared algorithm'. [Kelly97] proves that the use of a square region gives a slightly superior error detection performance. To stay conservative, AUGUR therefore assumes that the receiver uses a chi-squared algorithm.

2.3 Dealing with the unknown bias

In the case when a satellite error occurs both the probability of a position failure and an error detection depend on the unknown bias in the failing satellite. When the bias in a failing satellite grows, so do the biases in the position and test statistic, increasing the probabilities of a position error but also of detection. To enable the computation on the probability of HMI, some value will have to be substituted for the unknown bias to enable performance computation. Two strategies are known:

1. Substitute minimal detectable biases
2. Substitute worst case biases

They will be discussed in the next two paragraphs.

2.3.1 Minimal Detectable Biases

The minimal detectable bias (MDB) principle (e.g. [Sturza90,Leva96]) approaches the 'unknown bias' problem as follows. The TSO-C129 requirements state, that position errors have to be detected with a probability of at least 99.9%. For this requirement, the minimal detectable bias is defined as *the smallest satellite bias that can be detected with at least 99.9% probability*. Satellite biases smaller than the MDB will be detected with a lower probability.

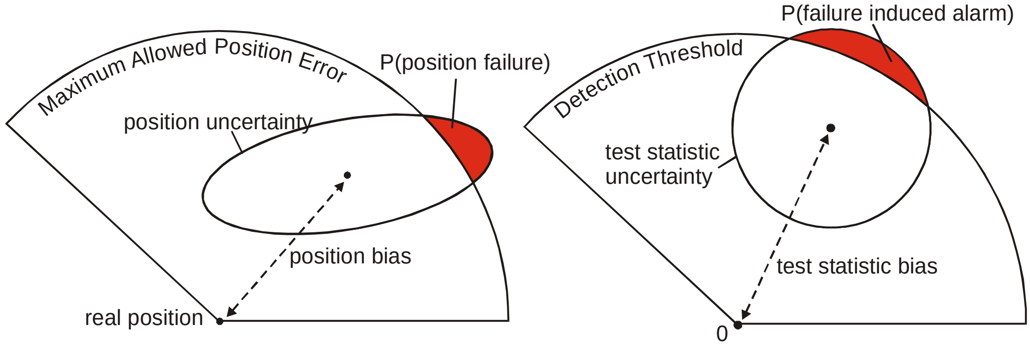

Figure 1a&b. The distribution of the position error and the least squares residual are both centred around the bias that is introduced by a failing satellite.

3 Often a so-called 'parity vector' is used, that has identical statistical properties, see [Brown92]

The situation is depicted symbolically in Figure 2a. The solid line in this figure represents the relation between the size of the position bias and the size of the test statistic bias. The ‘cloud’ that is drawn as an ellipse (although no real ellipsoid in practice) represents the uncertainty that is introduced by the presence of noise. It’s centre shifts towards the upper-right for a growing satellite bias. The MDB is the value of the satellite bias that causes exactly 99.9% of the noise cloud to lie beyond the error detection threshold.

It is important to realise that the MDB criterion implicitly assumes that every undetected satellite failure causes a position error. This effectively eliminates the need for a position error distribution! In reality, not all satellite errors result in HMI, as they don’t necessarily cause a sufficiently high position error. This would suggest that using the MDB gives conservative results. This would be perfectly true as long as the satellite bias indeed equals (or exceeds) the MDB. However, for some geometries satellite biases smaller than the MDB *might* cause higher HMI probabilities due to their reduced detectability. This means that for certain combinations of satellite geometry and satellite errors, the use of the MDB can lead to an underestimation of the probability of HMI.

2.3.2 Worst Case Satellite Biases

The possibility that the MDB approach could be underestimating the missed detection probability for certain satellite biases has been realised for some years now [Brown94,Lee95,Ober97]. Instead of only considering the detectability of a bias, as done in the MDB approach, these references all incorporate both the detectability *and* the chances of causing a position failure. They all propagate to solve explicitly for a *worst case bias* (WCB) that maximises the missed detection probability, as illustrated in Figure 2b. Unfortunately, the computation of the worst case bias for a particular geometry is computationally involved. The Approximate Radial error Protected (ARP) method [Chin92] has used Monte Carlo simulation to determine the worst case bias, but it is unclear whether the results can be applied for satellites constellations that differ from the one in the simulations.

2.3.3 Unknown biases and AUGUR

In the implementation of the AUGUR algorithm, it would have been appropriate to take the worst case bias approach because the MDB *might* lead to overly optimistic results. Unfortunately, the computational complexity of finding the worst case bias prevented the use of this approach, but this is most likely also true for current TSO receivers. Although the ARP method uses worst case biases and does most computations off-line, there are serious doubts whether its results are valid for general satellite constellations, making ARP a far from satisfactory alternative. Fortunately, when comparing the ARP outcomes to those obtained by MDB, it turned out that *for the given requirements and satellite noise levels*, MDB is more conservative than ARP is. This gives some confidence that receivers that use MDB might be more conservative. Consequently, it has been decided to use the MDB approach for AUGUR.

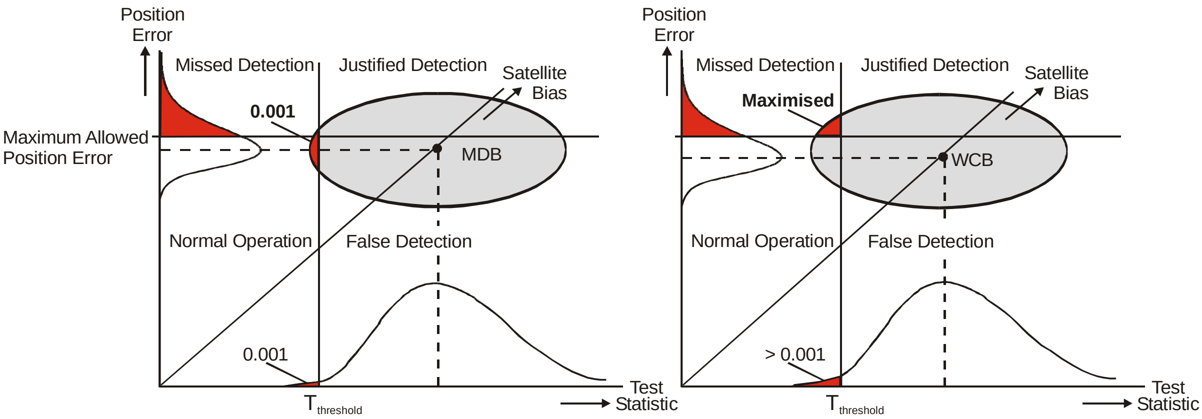

Figure 2a&b. *The position error and detection probabilities are a function of the unknown satellite bias B_{sat}. These figures show the situations when the unknown bias is replaced by the Minimum Detectable Bias (left hand side) or Worst Case Bias (WCB, right hand side).*

3. THE LEAST DETECTABLE SATELLITE

Often, the assumption is made that a satellite failure, when it occurs, is caused by the satellite that has the smallest error detection probability. In fact, many researchers have interpreted the following phrase out of [DO208] as a *demand* for this assumption:

> "*The integrity system shall meet the specified detection probability globally* <u>*at all times*</u> *for single satellite failures…*"

Moreover, the TSO-C129 receiver tests are performed by putting a ramp bias on this least detectable satellite and therefore seem to be based on this assumption. Of course, there is no reason why it should always be the least detectable satellite that fails, and it should be clear that assuming this leads to an overestimation of P_{HMI} and an underestimation of RAIM availability. Still, the assumption is widely used and applied [Brown90, Leva96, Sturza90, Lee95, Kelly97], and because of its conservatism, AUGUR incorporates it as well.

It remains to determine which of the satellites is 'least detectable'. All literature references seem to agree that the satellite with the highest slope in Figure 2 is the least detectable satellite, as it is this satellite that (in the noiseless case) has the worst ratio of position error to test statistic size. Although the use of a 'noiseless' approximation might not always be justified [Ober98], no other criterion seems to be available. AUGUR therefore uses the maximum slope over all satellites as a measure of geometric strength.

4. NON-ALGORITHMICAL CONCERNS

The AUGUR algorithm can obviously differ from receiver implementations. Its results, however, might also differ due to use of different parameter settings and to different satellite geometries as experienced by different receivers under different circumstances. An overview of such differences is presented in this section.

4.1 Parameters settings

Naturally, implementations can differ in the values chosen for the parameters that have been assumed fixed in the discussion in this paper:

- GPS/barometer range accuracy (standard deviation)
- failure rate of a GPS satellite/barometer

Obviously, higher values of these parameters would lead to a decrease in RAIM availability. The way of calibrating the barometer, and the use of filtering of the GPS pseudoranges to reduce noise, influence the performance of the algorithm as well. The standard deviation used by AUGUR is 33.3 m (from [DO-208]), while the satellite failure rate is set to three failures for the whole GPS constellation every 12 months (from [SPS]). This value is more conservative than the rate used in [DO-208], which assumes only one failure every 9 months.

4.2 Geometric differences

Although the GPS satellites possible to be observed are the same for all receivers that are operated at the same time at the same geographic position, they might still experience different measurement geometries because they track different satellites. Important factors to be considered are:

- mask angle
- number of satellites tracked for RAIM and positioning
- criteria used for subset selection: [DO208] proposes to select 5 satellites based on best accuracy considerations, while other sources [Graas93] propose a best integrity criterion instead. Other

criteria could be used as well.
- the possible use of barometric aiding

Note that the number of satellites tracked during operation might also change for different flight levels and bank angles.

5. THE AUGUR ALGORITHM IN SUMMARY

As shown above, RAIM algorithms can differ in a couple of ways. The algorithms can be either chi-squared or maximum residual based. Furthermore, two different methods for substitution of the unknown bias in case of a satellite failure can be exploited, based on either a minimal detectable or a worst case bias principle. Often, computations are simplified by assuming that it is always the least detectable satellite that fails, effectively representing the geometry by the slope of the worst case satellite only, although this results in conservative availability figures.

AUGUR uses the most conservative combination of design choices, and combines the chi-squared detection algorithm with the least detectable satellite failure assumption. Due to computational limitations, it uses the minimal detectable bias principle – which for the particular application is more conservative than the worst case bias figures used by the ARP algorithm. Because all choices in the algorithm design are described in many references from open literature, the final result is expected to be conservative without degrading the RAIM availability to an unreasonable extent, and therefore in line with both the JAA requirements and the end-user’s desires.

REFERENCES

[Brenner90] Brenner, M; “Implementation of a RAIM monitor in a GPS Receiver and an Integrated GPS”, ION, 1990
[Brown92] Brown, R.G.; “A Baseline RAIM scheme and a Note on the equivalence of three RAIM Methods”, ION, 1992
[Brown94] Brown, R. G., Kraemer, J. H., and Nim, G. C., “Comparison of FDE and FDI RAIM Algorithms for GPS”, Proceedings of the National Technical Meeting, Institute of Navigation, San Diego, CA, January 24-26, 1994, pp. 51-60
[Chin92] Chin, G.Y., J.H. Kraemer and R.G. Brown; “GPS RAIM: Screening Out Bad Geometries under Worst – Case Bias Conditions”, in *Global Positioning System: theory and applications Volume 2*, eds. B.W. Parkinson and J.J. Spilker Jr., American Institute of Aeronautics and Astronautics, 1996
[DO208] Minimum Operational Performance Standards for Airborne Supplemental Navigation Equipment Using Global Positioning System (GPS), RTCA Inc., July 1991, USA
[Graas93] Graas, F. van, and J. Farell; “Baseline Fault Detection and Exclusion Algorithm”, Proceedings of the 49th annual meeting of the institute of navigation, June 21-23, 1993
[Harriman98] Harriman, D.A.G. and P.B. Ober; “AUGUR: EUROCONTROL's Predictive RAIM tool for en-route”, these proceedings
[Kelly97] Kelly, R.J.; “Hypothesis Testing as applied to GPS Receiver Autonomous Fault Detection and Exclusion (FDE), Lecture Notes Presented at Ohio University Avionics Engineering Center
[Lee95] Lee, Y.C; “New techniques relating fault detection and exclusion Performance to GPS Primary Means Integrity Requirements”, The 8th international technical meeting of the ION, September 1995
[Leva96] Leva, J.L., M. Uijt de Haag and K. Van Dyke; “Performance of Standalone GPS”, *in Understanding GPS: Principles and Applications*, Edited by E.D. Kaplan, Artech House, 1996
[Ober97] Ober, P.B.; “Ways to improve RAIM/AAIM availability using position domain performance computations”, Proceedings of the National Technical Meeting of the Institute of Navigation, Santa Monica, January 1997
[Ober98] Ober, P.B.; “RAIM Performance: how algorithms differ”, The proceedings of the 11th International Technical Meeting of The Satellite Division of The Institute of Navigation, ION GPS-98, September 15-18, 1998, Nashville, Tennessee
[SPS] Global Positioning System Standard Positioning Service Signal Specification, 2nd edition, June 2, 1995
[Sturza88] Sturza, M.A.; “Navigation System Integrity Monitoring using redundant measurements”, Navigation, Vol. 35, No. 4, Winter 1988-89
[Sturza90] Sturza, M.A. and A.K. Brown; “Comparison of fixed and variable threshold RAIM algorithms”, proceedings of ION, pp. 437-443, 1990
[TGL-2] Temporary Guidance Leaflet no. 2 (formerly AMJ 20X-2), June 1997, Preliminary Draft, Joint Aviation Authorities
[TSO-C129] Airborne Supplemental Navigation Equipment Using the Global Positioning System (GPS), Department of Transportation, FAA Aircraft Certification Service, 12 October 1992, Washington DC, USA