AUTONOMOUS INTEGRITY MONITOR PERFORMANCE

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BIOGRAPHY

Bastiaan Ober has 4 years of experience in radio navigation. He received his M.Sc. in Electrical Engineering from Delft University of Technology in 1993. Areas of experience include the influence of multipath on GPS positioning, carrier phase differential GPS, ambiguity resolution and integrity monitoring. He is currently working as a Ph.D. student at Delft University of Technology, doing research on integrity monitoring algorithms for integrated navigation systems.

ABSTRACT

System Integrity concerns “the ability of a system to provide timely warnings when the system should not be used because it possibly does not operate within specifications”. An autonomous integrity monitor (AIM) measures this ability. In the navigation world, AIM is traditionally identified with ‘error detection’. Given the definition of integrity, AIM should rather be identified with ‘error detectability’ instead. This basic misunderstanding sometimes complicates insight in AIM performance. Moreover, current AIM formulations address detection of measurement errors rather than position errors. As a result, AIM performance is often specified in the power of detecting measurement errors, instead of the position errors on which Required Navigation Performance parameters are based.

Rather than elaborating on AIM mathematics, this paper shows the concepts behind the computation of AIM performance in the position domain, and explains which assumptions have to be made. Understanding of these concepts, and awareness of the assumptions, will provide the reader with a suitable framework to judge the value of AIM performance studies with their inherent limitations.

1. INTRODUCTION

Users of navigation systems need to be warned when the system can not be guaranteed to be working within the required specifications. Integrity is the ability of a navigation system to provide the user with such warnings, and is in fact nothing but the power of the system to detect errors.
An integrity monitoring system usually consist of two parts [Ober96] (see figure 1):

- The *error detector* will warn the user in case of system errors. It computes a test statistic T that grows with the mutual inconsistency of the navigation signals. If the test statistic surpasses a certain threshold $T_{Threshold}$, an error is detected and the user is shown a red light.

- The *error detectability* monitor determines whether sufficient detection power is available. If the system lacks detection power, and the probability that a position error remains undetected (P_{MD}) becomes too high, the user is shown a yellow light. The yellow light is also shown when the probability that a false detection is generated (P_{FD}) is unacceptably large.

When we consider the definition of integrity, it should be clear that the error detectability monitor is the actual integrity monitor. However, because 'error detectability' is only meaningfully defined in combination with a certain error detector, the latter is generally assumed to be an integral part of the monitor as well.

In this paper, we will focus on Autonomous Integrity Monitoring (AIM), a technique that uses redundancy of the signals used in the position computation to extract information on error characteristics. Two different variants exist: Receiver AIM (RAIM) uses signals from one system (usually GPS) only, Aircraft AIM (AAIM) includes information from other sensors as well. When all sensors are fully integrated and used in the position solution, there is no fundamental difference, and we will use the generic term 'AIM' in this paper.
After the introduction of the system model in section 2, we will discuss computation of three types of information that the navigation system should provide: position information in section 3, error information in section 4, and integrity information in section 5. Section 6 concludes with a short summary.

2. SYSTEM DESCRIPTION

We will assume that the relation between the measurements that the navigation system provides, and the actual position of the user, is given by an overdetermined linear regression model with *n* measurements in *m* unknowns (*n*>*m*):

$$\vec{z} = H \cdot \vec{x} + \vec{v} \tag{2.1}$$

in which:
$\vec{z}$: *n*-vector of measurements
H : $n \times m$ observation matrix
$\vec{x}$: *m*-vector of unknowns (usually position, clock bias)
$\vec{v}$: *n*-vector with independent noise and biases in the measurements
A usual assumption is that the noise is normally distributed with mean $\vec{\mu}_v$ and covariance R_v:

$$\vec{v} \sim \mathrm{N}(\vec{\mu}_v, R_v) \tag{2.2}$$

When the mean of the noise $\vec{\mu}_v$ is nonzero, it will be called the *measurement bias*.

We would like to extract three types of information from the navigation system:

- the unknown position (positioning)
- the possible presence of position errors (error detection)
- the detectability of position errors (AIM performance)

We will discuss computation of each of these in the coming sections.

3. POSITION ESTIMATION

When the noise is normally distributed, the estimated position is simply computed by Weighted Least Squares (WLS):

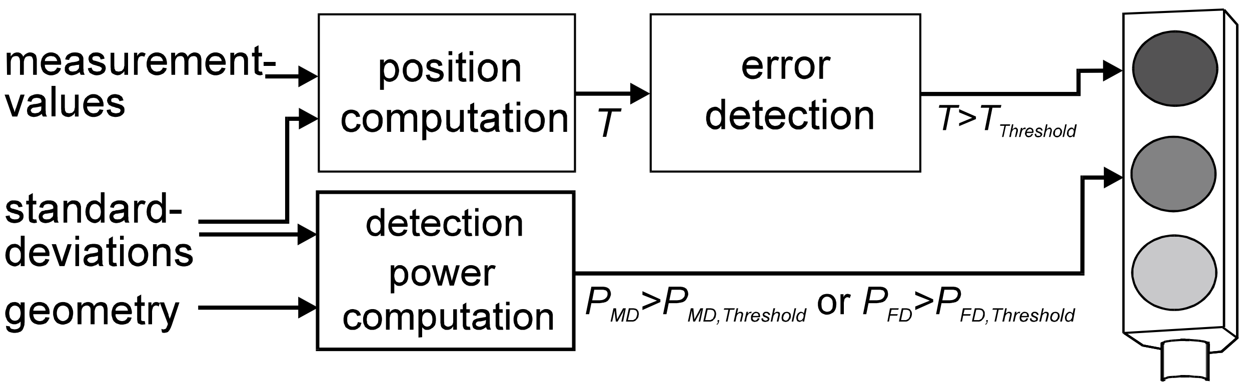

Figure 1. An Integrity Monitor with its input and output signals

$$\hat{\vec{x}}_{LS} = N\vec{z} \tag{3.1}$$

with $N = (H^T R_\nu^{-1} H)^{-1} H^T R_\nu^{-1}$. Due to noise and biases in the measurements, $\hat{\vec{x}}_{LS}$ will differ from the exact position. This is acceptable as long as the error in the position remains within the boundaries set by the Required Navigation Performance (RNP). A warning should be generated when the position error might be out of bound. Therefore, when the position error is defined as

$$\Delta\hat{\vec{x}}_{LS} = \vec{x} - \hat{\vec{x}}_{LS} \tag{3.2}$$

and the error allowed by the RNP is denoted by $RNP_{\Delta\hat{x}}$, we would like to know which of the following hypotheses is true:

$$\begin{aligned} &H_0(no\ \ position\ \ error): \left\|\Delta\hat{\vec{x}}_{LS}\right\| \le RNP_{\Delta\hat{x}} \\ &H_1(position\ \ error): \quad \left\|\Delta\hat{\vec{x}}_{LS}\right\| > RNP_{\Delta\hat{x}} \end{aligned} \tag{3.3}$$

When there is an excessive probability that H_1 is true, it is likely that there is an error in the system, and a red light should be shown to the user.

To decide among these hypotheses we will have to find some test statistic T that gives an indication about the position error. This test statistic can be used together with a threshold $T_{Threshold}$ in a decision scheme that will look something like this:

$$\begin{aligned} &T \le T_{Threshold} \Rightarrow H_0 \ \ is\ \ accepted \\ &T > T_{Threshold} \Rightarrow H_1 \ \ is\ \ accepted \end{aligned} \tag{3.4}$$

Using a decision criterion such as (3.4), it can happen that we accept the wrong hypothesis. The two possible decision errors that can be made are called missed detection (accepting H_0 unjustly) and false detection (accepting H_1 unjustly). The probabilities of missed and false detection (P_{MD} and P_{FD}) determine how well position errors can be detected based on a decision involving T. When these probabilities become too high, the user can not trust the error detection scheme sufficiently, and an orange light should be issued.

Therefore, P_{MD} *and* P_{FD} *are the parameters that measure AIM performance*, and we would like to compute them along with the position and error estimates. We will discuss this computation in section 5. First, we have to consider the choice of the test statistic T.

4. ERROR DETECTION USING THE LEAST SQUARES RESIDUAL

The position error depends on the unknown noise and biases in the system. When there is redundancy in the system, it is possible to estimate the noise and biases $\vec{\nu}$ from the least squares residual [Rao95]:

$$\hat{\vec{\nu}}_{LS} = D\vec{z} \tag{4.1}$$

with $D = I - H(H^T R_\nu^{-1} H)^{-1} H^T R_\nu^{-1}$.

The estimated noise is a natural basis for error detection[1], and in this section we will therefore investigate the relation between the least squares residual and the position error.

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[1] In fact, all possible test statistics are necessarily a function of the least squares residual, see for example [Lehmann86]

The first and most important relation between residual and position error is their orthogonality, that is due to the orthogonal projection that the least squares scheme performs. The spaces in which the position error and the residual lie are orthogonal in the sense that:

$$\hat{\vec{v}}_{LS}^{T} R_{v} \Delta\hat{\vec{x}}_{LS} = 0 \tag{4.2}$$

which can readily be verified by substituting (3.2) and (4.1) in (4.2). This implies that the noise and bias vector $\vec{v}$ can be decomposed in two orthogonal components: one that influences *only* the position error, and one that influences *only* the residual:

$$\vec{v} = N\vec{v}_{pos} + D\vec{v}_{res} \tag{4.3}$$

Moreover, due to the orthogonality and the independent normal distribution of the elements of $\vec{\nu}$, $N\vec{v}_{pos}$ and $D\vec{v}_{res}$ are statistically independent [Johnson72], and contain no mutual information [Jones79].

The situation is depicted in figure 2 and figure 3 (for simplicity, we used 2 measurements in only 1 unknown). The position error space and the residual space are depicted as orthogonal subspaces of the 'measurement noise' space. The two-dimensional Gaussian distribution of the measurement noise, and the one-dimensional Gaussian distributions of position error and residual are drawn symbolically as well.

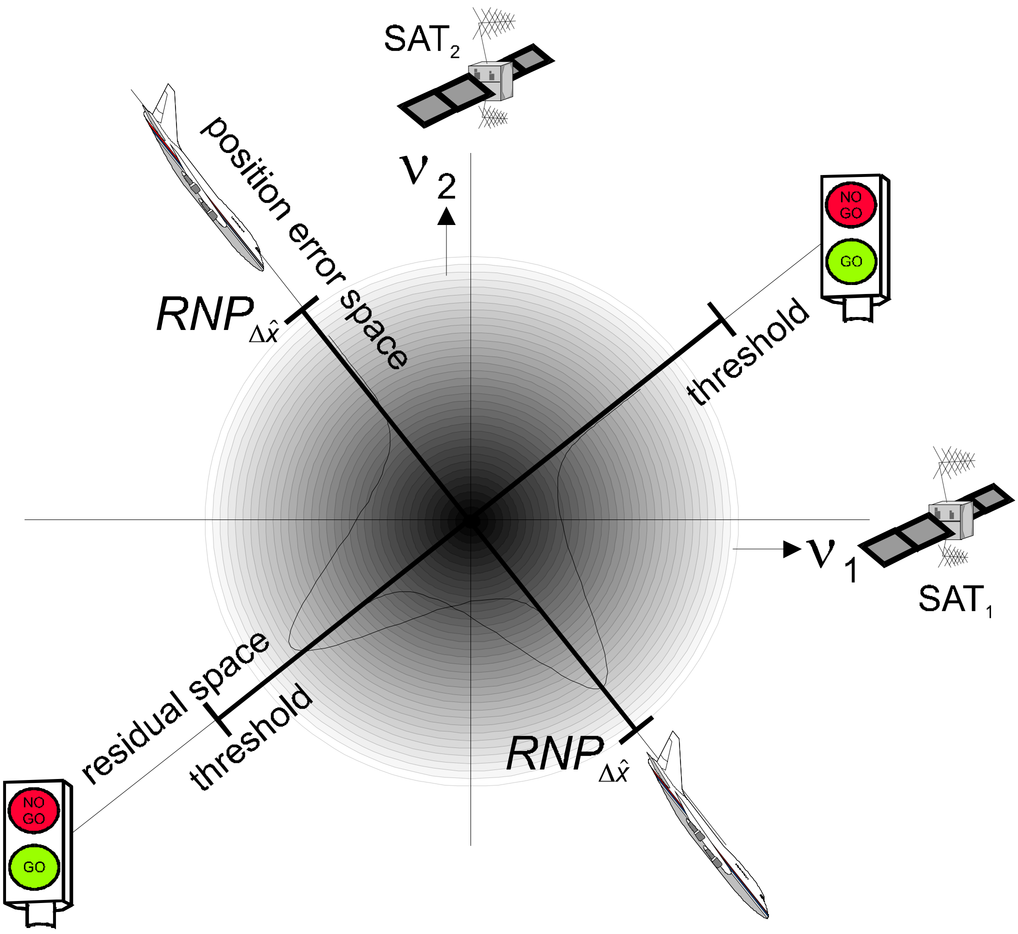

Figure 2. Noise during normal system operation: measurement noise is two-dimensional Gaussian, position error and residual are one-dimensional Gaussian

The inevitable conclusion seems to be rather discouraging: the residual doesn't tell us anything about the position error. Still, we claim it can be used in error detection. To understand why, we will have to distinguish between two 'types' of errors. The first type is the one that occurs during normal operation of the system, and is caused by *noise* only. The second type is the one that is caused by anomalies in the system, and is caused by a combination of *noise and biases*.

During normal operation, each measurement source provides essentially unbiased measurements[2] that contain only 'random' errors. This was the situation of figure 2. When one of the measurement sources starts to operate according to a model that deviates from the assumed one, its measurement errors will either get biased, more noisy, or both. When looking at a single sample, these effects can not be distinguished, and we will therefore consider the occurrence of a bias only.
Unlike the noise, that is stochastic, biases are deterministic in character. In general, they will have components in both the 'position error space' and the 'residual space'. These components will be perfectly correlated. This means that it will be possible to detect such biases from the residual, as long as they are significant with respect to the noise.

Detection of a measurement bias from noisy measurements is a problem that has been extensively studied in literature [Basseville93]. It is usually formulated as the hypotheses testing problem of deciding among:

$$\begin{aligned} &H_0(\text{no measurement bias}): \vec{\mu}_\nu = \vec{0} \\ &H_1(\text{measurement bias}): \quad \vec{\mu}_\nu \neq \vec{0} \end{aligned} \tag{4.4}$$

Note that this is not the same detection problem as the one that we started out with! This is something

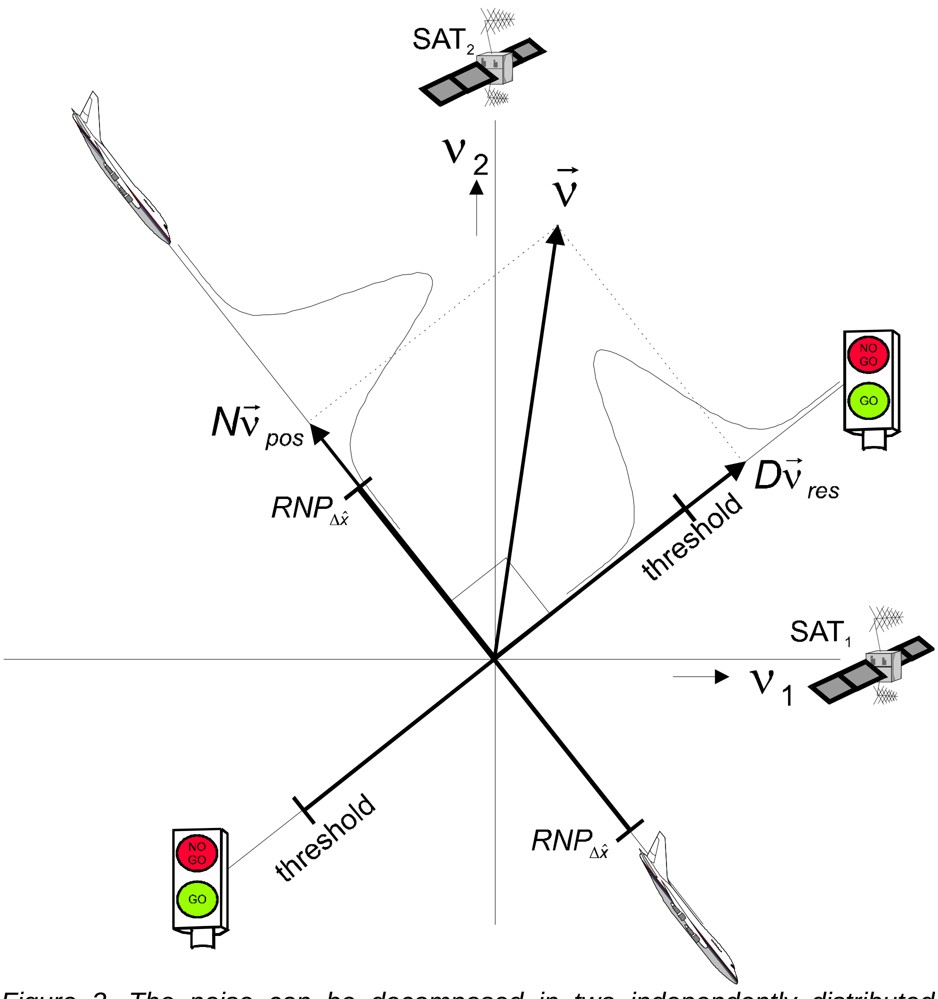

Figure 3. The noise can be decomposed in two independently distributed components that influence position only ($N\vec{\nu}_{pos}$) and residual only ($D\vec{\nu}_{res}$)

that is often overlooked, and sometimes the probabilities of missed and false detection belonging to problem (4.4) are presented as the performance parameters of AIM [Leva96, Brown96]. However, RNP explicitly requires that AIM performance is expressed in terms of positioning errors, and should therefore relate to (3.3). We will show how we can compute this performance in the next section.

5. AIM PERFORMANCE (ERROR DETECTABILITY)

For a given measurement bias, it is relatively easy to compute the probability we will detect it, and the probability that it will cause a position error. The larger the bias becomes, the easier it becomes to detect it, but the likelier it gets that it causes a position error. Unfortunately, we will have to work with limited knowledge on the measurement bias. This section will focus on the use of this knowledge to compute upperbounds on the probability of missed detection of a position error[3], and will clarify which assumptions have to be made to be able to do so.

The probability that a bias $\vec{\mu}_\nu^{(i)}$ causes a missed detection is the probability of a coincidence of three events:

1. a bias $\vec{\mu}_\nu^{(i)}$ is present
2. $\vec{\mu}_\nu^{(i)}$ causes a position error
3. $\vec{\mu}_\nu^{(i)}$ does not cause a detection

When we introduce the following notations:

probability that a bias $\vec{\mu}_\nu^{(i)}$ occurs: $P_{bias}(\vec{\mu}_\nu^{(i)}) = P(\vec{\mu}_\nu = \vec{\mu}_\nu^{(i)})$

probability that $\vec{\mu}_\nu^{(i)}$ causes a position error: $P_{pos\_error}(\vec{\mu}_\nu^{(i)}) = P(\left\| \Delta\hat{\hat{x}}_{LS}(\vec{\mu}_\nu^{(i)}) \right\| > RNP_{\Delta\hat{x}})$

probability that $\vec{\mu}_\nu^{(i)}$ doesn't cause a detection: $P_{no\_detection}(\vec{\mu}_\nu^{(i)}) = P(T(\vec{\mu}_\nu^{(i)}) \le T_{Threshold})$

and exploit the independence of P_{pos_error} and $P_{no\_detection}$, the probability of missed detection due to a bias $\vec{\mu}_\nu^{(i)}$ can be written as:

$$P_{MD}(\vec{\mu}_\nu^{(i)}) = P_{bias}(\vec{\mu}_\nu^{(i)}) \cdot P_{no\_detection}(\vec{\mu}_\nu^{(i)}) \cdot P_{pos\_error}(\vec{\mu}_\nu^{(i)}) \tag{5.1}$$

Because the bias $\vec{\mu}_\nu^{(i)}$ will be unknown, we can never compute exact values of P_{MD}. Often, all we can do is determining its worst case value.

Unfortunately, no full knowledge of $P_{bias}(\vec{\mu}_\nu^{(i)})$ can be assumed available: we simply don't know how likely it is for a certain bias to occur. Usually, limited knowledge *can* be assumed to be present on the probability that a certain type (class) of biases occurs. The most well known class is the 'single failure in measurement *i* ' class, in which it is assumed that a bias occurs in measurement *i* only. In this class, every bias vector therefore has the form:

$$\vec{\mu}_\nu^{(i)} = \begin{bmatrix} 0 \cdots 0 & \mu_i & 0 \cdots 0 \end{bmatrix}^T \tag{5.2}$$

From the design parameters of GPS, the probability of failure of a single satellite $P_{single\_failure}$ can be deduced [Shively93], and for this particular class of biases we can therefore set

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[2] Note that this is not exactly true for some error sources such as Selective Availability. We plan to address this problem in a future paper.

[3] AIM performance also includes the false detection probability, for which the exact same line of reasoning is applicable. We will therefore limit the discussion to missed detections only.

$$P_{bias}(\vec{\mu}_\nu^{(i)}) = P_{single\_failure} \tag{5.3}$$

This leaves us with the maximization of the missed detection probability over all possible biases in measurement *i*. This maximum is reached when the product of P_{pos_error} and $P_{no\_detection}$ is as large as possible, and can be found by standard function maximization algorithms, as has been discussed recently in [Lee95, Kraemer97, Ober97]. These algorithms maximize the probability content of the 'missed detection' area in figure 4, over all biases $\vec{\mu}_\nu^{(i)}$ in the class.

Although 'the single failure in measurement *i* ' is the most widely used class, the same approach can be used for other classes of bias, for example:

- simultaneous failures in measurement *i* and *j*
- all biases that obey $\|\vec{\mu}_\nu\| \leq \alpha$ (multipath and maybe interference could be modeled this way)
- no failure (normal system operation)
- combinations of the classes above

Note that the 'no failure' class only contains the bias vector $\vec{\mu}_\nu^{(i)} = 0$, which makes explicit maximization unnecessary.

When we use the notation

$$P_{MD}(M_i) = \max_{\vec{\mu}_\nu^{(i)} \in M_i} \left\{ P_{no\_detection}(\vec{\mu}_\nu^{(i)}) \cdot P_{pos\_error}(\vec{\mu}_\nu^{(i)}) \right\} \tag{5.4}$$

for the maximum (conditional) probability that a bias in class M_i causes an undetected position error, we can finally compute a worst case 'overall' probability of a missed position error detection, by summing over all classes of biases that should be considered[4]:

$$P_{MD} = \sum_i P_{MD}(M_i) \tag{5.5}$$

This value should then be compared to the missed detection probability that is specified by the Required Navigation Performance standard.

6. CONCLUSION

In this paper, we have discussed the computation of the probability that a position error remains undetected, which is the main performance parameter of an Autonomous Integrity Monitor. We have also shown that this probability differs from the often-used probability that a measurement bias remains unnoticed.

One of the goals of this paper has been, to make all necessary assumptions to compute AIM performance explicit. In short, these assumptions are:

- during normal system operation all measurements are unbiased, that is, $\vec{\mu}_\nu = \vec{0}$
- all biases that can occur belong to certain mutual exclusive classes, that have an (approximately) known probability of occurrence
- the 'worst case' bias is taken to be representative for the whole class, as long as no additional knowledge on distributions within a class is available

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[4] assuming that the classes of biases are mutual exclusive, that is, each bias can belong to at most one class

It is clear that any figures for missed and false detection probabilities heavily rely on the correctness of all these assumptions. On the one hand, worst case biases are taken, which might give pessimistic values for P_{MD} and P_{FD} . On the other hand, we should take care that the probability that a bias in a certain class occurs is not underestimated, and that no classes of biases are left out of the analysis.

Understanding of the inherent limitations will enable fair judgment of the value of studies into AIM performance, and should provide a framework for setting up new studies as well.

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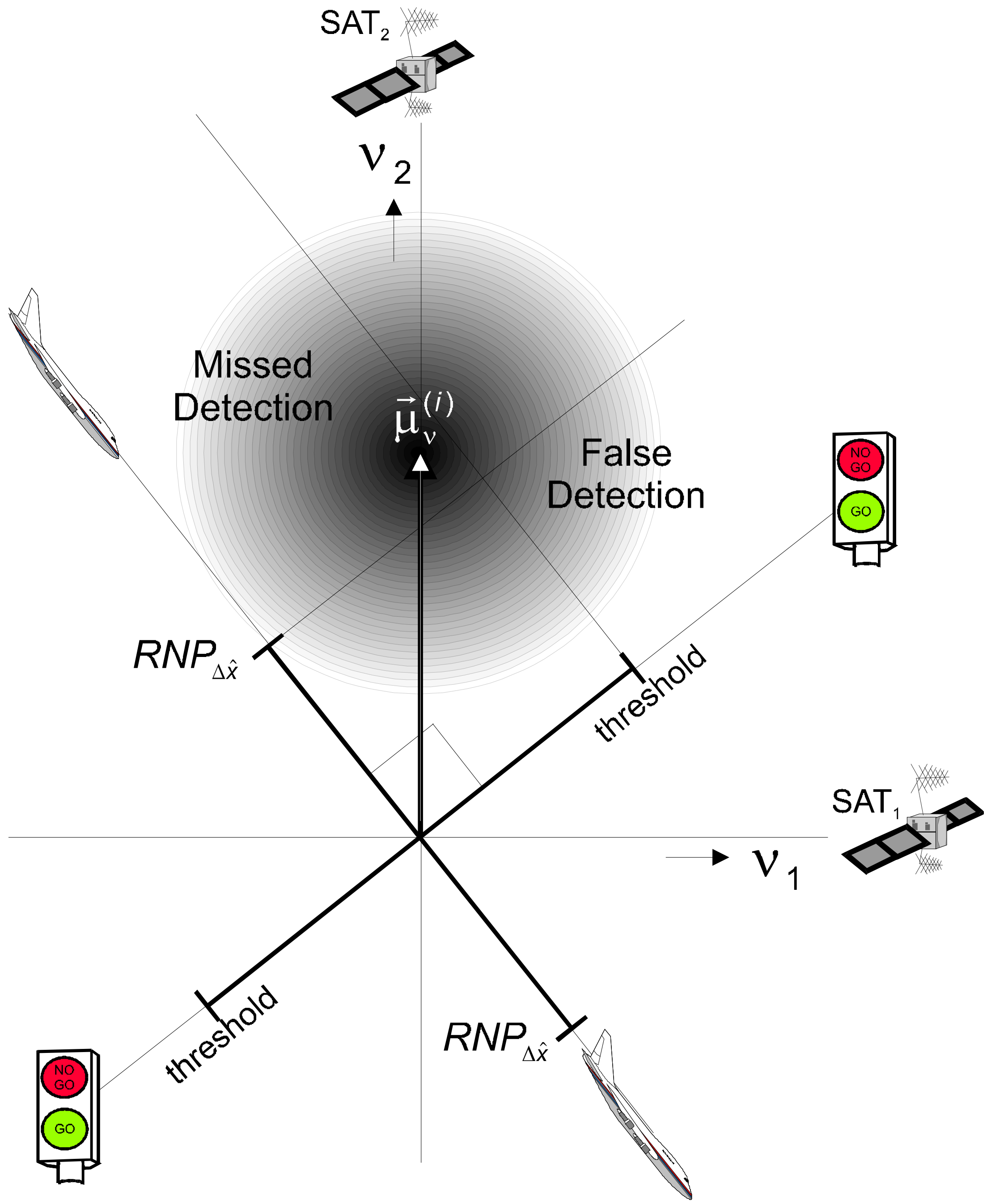

Figure 4. The missed and false detection probabilities are the probability contents of subspaces of the measurement noise space, and depend on the value of the measurement bias

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